

**A STUDY OF ELASTIC WAVES IN
ELECTRO-MAGNETO THERMOELASTIC MATERIALS**

**A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF DOCTOR
OF PHILOSOPHY**

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**A STUDY OF ELASTIC WAVES IN
ELECTRO-MAGNETO THERMOELASTIC MATERIALS**

BY

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Submitted

**In partial fulfillment of the requirement of the Degree of Doctor of
Philosophy in Mathematics of Mizoram University, Aizawl.**

**DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE
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CERTIFICATE

This is to certify that the thesis entitled “A study of Elastic waves in electro-Magneto Thermoelastic Materials” submitted by Ms. Shrabanika Boruah for the award of the degree of Doctor of Philosophy (Ph.D.) in Mathematics, is a bonafide record of the original research carried out by her under my supervision. She has been duly registered and the thesis is worthy of being considered for the award of the Ph.D. degree.

I, the undersigned has declared that this research work has been done under my supervision and has not been submitted for any degree of any other universities.

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Supervisor

DECLARATION

Mizoram University

July, 2025

I, **Ms. Shrabanika Boruah**, hereby declare that the subject matter of this thesis is the record of work done by me, that the contents of this thesis did not form basis of the award of any previous degree to me or to do the best of my knowledge to anybody else, and that the thesis has not been submitted by me for any research degree in other University/Institute.

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Dated:

Place: Aizawl

(SHRABANIKA BORUAH)

PREFACE

The present thesis entitled “**A Study of Elastic Waves in electro-Magneto Thermoelastic Materials**” is an cumulation of the research carried out by me under the supervision of Prof. S. Sarat Singh, Department of Mathematics & Computer Science, Mizoram University, Aizawl - 796 004, Mizoram, India.

This thesis consists of problems associated with the propagation of elastic waves in generalized magneto thermoelastic materials and generalized magneto micropolar thermoelastic materials. The phenomena of reflection and refraction of elastic waves are discussed using relevant boundary conditions in the materials. It consists of six chapters. The first chapter is the general introduction of the thesis which includes the basic definitions, thermoelasticity, reflection and refraction of waves, scope of the study and review of literature.

Second chapter deals with the problem of reflection of homogeneous plane waves from the free surface of a generalized magneto thermoelastic half-space. The phase speeds of corresponding to longitudinal, transverse and thermal waves are obtained and are independent of angle of propagation. The amplitude and energy ratios corresponding to the reflected waves are obtained theoretically and numerically with the help of boundary conditions. The effects of magnetic and thermal parameters on the reflected waves are examined. The results hold the conservation law of energy. Various special cases are derived for this problem.

Third chapter studied the three dimensional problem of wave propagation in generalized magneto thermoelastic medium. The eigenvalue approach is adopted to solve the vector matrix differential equation which represents the equation of motion for the generalized magneto thermoelastic materials. Components of displacements and temperature field are analyzed theoretically and computed numerically with the help of appropriate boundary conditions. The effect of magnetic and relaxation time parameters on the propagation of waves are critically examined.

Chapter 4 discusses the problem of reflection of homogeneous plane waves from

the free surface of generalized magneto micropolar thermoelastic half-space using modified Ohm's law and generalized Fourier law. There are five waves propagated in the medium and these waves are longitudinal, transverse, micropolar, thermal and another wave due to magnetic field. The phase velocities of these waves are obtained analytically and numerically. Using appropriate boundary conditions, the amplitude and energy ratios corresponding to the reflected waves are obtained analytically and numerically. Magnetic and thermal effects on the reflected waves are investigated and found the conservation law of energy in all cases.

Chapter 5, we investigated the problem of reflection and refraction of homogeneous plane waves from the free surface of generalized magneto micropolar thermoelastic half-space using modified Ohm's law and generalized Fourier law. The phase velocities of these waves are obtained analytically and numerically. The amplitude and energy ratios corresponding to the reflected and refracted waves are obtained analytically and numerically with the use of suitable boundary conditions. Magnetic and thermal effects on the reflected and refracted waves are investigated and we obtain conservation law of energy.

Chapter 6 is summary and conclusions of the thesis.

A list of references has been given at the end.

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GLOSSARY

Term	Explanation
Stress	Force per unit area.
Strain	Deformation due external force.
Equation of motion	Motion of particles in the continua.
Wave	Disturbance that transfers energy from one position to another.
Seismic Waves	Wave due to Earthquakes or explosion inside the Earth crust.
Longitudinal wave	Particles of the medium vibrate parallel to the direction of the wave.
Transverse wave	Particles of the medium vibrate perpendicular to the direction of the wave.
Temperature wave	Wave due to temperature field.
Micropolar wave	Wave due to rotations of micro-structure elements.
Wavelength	The distance between two corresponding points on consecutive waves.
Frequency	The number of waves that pass a point in a given amount of time.
Amplitude	The maximum displacement of a wave from it's resting position.
Phase velocity, V_i	The speed at which the wave propagates through a medium.
Incident wave	The wave travelling towards the barrier.
Reflected wave	The wave which changes direction when it encounters a barrier.
Refracted wave	The wave which passes through one medium to another through a barrier.
Amplitude ratio, Z_i	The ratio of amplitude of reflected or refracted wave to that of incident wave.
Energy ratio, E_i	The ratio of energy corresponding to reflected or refracted wave to that of incident wave.

Chapter 1

Introduction and Review of Literature

1.1 Base vectors

An arbitrary vector in three-dimensional space can be written as a linear combination of any three linearly independent vectors. As a special case, the base vectors $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ of a rectangular Cartesian Coordinate system are taken to be constantly orthonormal vectors which form a right handed system. The notions of linear independence and right handedness can be written in the mathematical form

$$(\mathbf{e}_1 \times \mathbf{e}_2) \cdot \mathbf{e}_3 > 0 \quad (1.1)$$

where $\cdot$ denotes the usual scalar dot product operation and $\times$ denotes the usual cross product operation between two vectors. Also, these base vectors are orthonormal vectors which indicate that they satisfy the following restrictions

$$\begin{aligned} \mathbf{e}_1 \cdot \mathbf{e}_1 &= 1, \mathbf{e}_1 \cdot \mathbf{e}_2 = 0, \mathbf{e}_1 \cdot \mathbf{e}_3 = 0, \\ \mathbf{e}_2 \cdot \mathbf{e}_1 &= 0, \mathbf{e}_2 \cdot \mathbf{e}_2 = 1, \mathbf{e}_2 \cdot \mathbf{e}_3 = 0, \\ \mathbf{e}_3 \cdot \mathbf{e}_1 &= 0, \mathbf{e}_3 \cdot \mathbf{e}_2 = 0, \mathbf{e}_3 \cdot \mathbf{e}_3 = 1. \end{aligned} \quad (1.2)$$

Moreover, it follows that these base vectors satisfy the additional equations

$$\mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3, \quad \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2, \quad \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1. \quad (1.3)$$

1.1.1 Components of a vector

The components $\{a_1, a_2, a_3\}$ of a general vector, $\mathbf{a}$ can be used together with the base vectors $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ to express $\mathbf{a}$ in the form

$$\mathbf{a} = a_1 \mathbf{e}_1 + a_2 \mathbf{e}_2 + a_3 \mathbf{e}_3.$$

Using the dot product, it follows that the components $\{a_1, a_2, a_3\}$ of $\mathbf{a}$ are the projections of $\mathbf{a}$ in the directions of the base vectors, $a_1 = \mathbf{a} \cdot \mathbf{e}_1$, $a_2 = \mathbf{a} \cdot \mathbf{e}_2$, $a_3 = \mathbf{a} \cdot \mathbf{e}_3$.

1.1.2 Free Indices

Indices that appear only once in a given term are known as free indices. For our purposes, each of these free indices will take the values 1, 2 and 3. For example, i is a free index in each of the following expressions:

$$(x_1, x_2, x_3) = x_i, \quad \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} = \mathbf{e}_i.$$

The free index i in the above expressions refers to the group of three quantities defined by i taking the values 1, 2 and 3.

1.1.3 Repeated Indices

Indices that appear twice or more in a given term are known as repeated indices. For example i and j are repeated indices in the following expressions:

$$x_i \mathbf{e}_i, \quad a_j \mathbf{e}_j, \quad a_i b_i.$$

It may be noted that the notion of a term is each group of symbols which are separated by a + sign or a - sign or an = sign.

1.1.4 Einstein Summation Convention

When an index appears as a repeated index in a term, then that index is understood to take on the values $(1, 2, 3)$ and the resulting terms are summed. For example, the vectors $\mathbf{x}$ and $\mathbf{a}$ can be expressed in the forms

$$\mathbf{x} = x_i \mathbf{e}_i = x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + x_3 \mathbf{e}_3, \quad \mathbf{a} = a_j \mathbf{e}_j = a_1 \mathbf{e}_1 + a_2 \mathbf{e}_2 + a_3 \mathbf{e}_3.$$

These repeated indices are also known as dummy indices since their replacement by any other letter, not appearing as a free index and also not appearing as another repeated index, do not change the meaning of the term in which they occur. For examples, in the expressions

$$x_i \mathbf{e}_i = x_j \mathbf{e}_j \text{ \& } a_i b_i = a_j b_j, \quad (1.4)$$

$x_i \mathbf{e}_i$ with $x_j \mathbf{e}_j$ and $a_i b_i$ with $a_j b_j$ have same meaning. It is important to note that the same free indices must appear in each term in an vector equation

$$\mathbf{c} = \mathbf{a} + \mathbf{b} \quad (1.5)$$

can be written in index form in terms of the components of $\mathbf{a}, \mathbf{b}, \mathbf{c}$ as

$$c_i = a_i + b_i.$$

1.1.5 Kronecker Delta

The Kronecker delta symbol, δ_{ij} is defined by

$$\delta_{ij} = \mathbf{e}_i \cdot \mathbf{e}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

1.1.6 Permutation symbol

The permutation symbol, ε_{ijk} is defined by

$$\varepsilon_{ijk} = \mathbf{e}_i \times \mathbf{e}_j \cdot \mathbf{e}_k = \begin{cases} 1 & \text{if } (i, j, k) \text{ is an even permutation of } (1, 2, 3) \\ -1 & \text{if } (i, j, k) \text{ is an odd permutation of } (1, 2, 3) \\ 0 & \text{if at least two of } (i, j, k) \text{ are equal.} \end{cases}$$

1.1.7 Cubical Dilatation

The cubical dilatation, Σ is connected in terms of the components of displacement, (u, v, w) by the following equation

$$\Sigma = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}. \quad (1.6)$$

1.2 Continuum concept

A continuum model assumes that the particles of the object completely fill the space and provides a sufficiently accurate description of the properties of matter on length scales much greater than that of inter-atomic distances. The theory of continuum body is built up on basic concepts of stress, strain, motion and constitutive relations based upon the laws of conservation of mass, linear momentum, moment of momentum, energy and angular momentum. The constitutive relations characterize the mechanical and thermal responses of a material, while the basic conservation laws give the common features of all mechanical phenomena irrespective of the constitutive relations.

Continuum Mechanics is a branch of mechanics that deals with kinematics and mechanical behaviour of materials modelled as continuous mass rather than discrete particles. If a body contains sufficiently large number of molecules, so that the distance between two neighbouring molecules are negligible in comparison with the dimensions of the body, then the body is called continuous body and it behaves in

accordance with the law of mechanics.

1.2.1 Deformation

When an elastic continuum body, which is in equilibrium, is subjected to some external forces, then the body undergoes deformation. In this process, the particles act against internal resistant forces of the material, that exist even in the absence of external forces. Let $\mathbf{R} = X_1\hat{i} + X_2\hat{j} + X_3\hat{k}$ and $\mathbf{r} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$ be the position vector of a particle in the original body, V_0 at time t_0 and deformed body, V at time $t_0 + t$ respectively. This vector, $\mathbf{R}$ refers the position of particle, while vector, $\mathbf{r}$ describes final position of the particles after deformation. Since $\mathbf{R}$ denotes a generic particle initially inside a volume V_0 , the vector $\mathbf{r}$ may be written as

$$\mathbf{r} = \mathbf{r}(\mathbf{R}, t) \quad (1.7)$$

represents the deformation (motion) of all the particles in V_0 as shown in Figure 1.1.

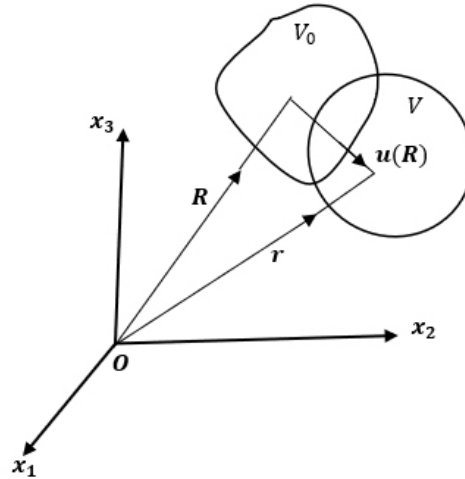


Figure 1.1: Geometry of deformation

The displacement vector $\mathbf{u}(\mathbf{R})$ is the difference between $\mathbf{r}$ and $\mathbf{R}$, and is a function of position in the undeformed state. In component form, we may write

$$r_i = x_i = x_i(X_1, X_2, X_3, t), \quad i = 1, 2, 3, \quad (1.8)$$

where $x_i = (x_1, x_2, x_3)$.

Since there is one to one correspondence between the $\mathbf{R}$ and $\mathbf{r}$, the inverse relation also holds as

$$\mathbf{R} = \mathbf{R}(\mathbf{r}, t). \quad (1.9)$$

This deformation is caused due to an external force, which is either a *body force* that acts at a distance within a body or between bodies or a *surface force* which acts upon a surface element of the body regardless of whether that element is a part of the boundary surface or an arbitrary element of surface within the body.

1.2.2 Stress and Strain

The force per unit area set up inside the body to resist the deformation is called *stress*. Let ΔS be a surface element of a body with volume V as shown in Figure 1.2. Let the force acting on the surface element ΔS be denoted by $\mathbf{F}$. The stress vector $\mathbf{T}$ which represents the surface force per unit area acting at the point (x_i) across the surface element with outward unit normal vector $\hat{\mathbf{n}}$ is given by (Sokolnikoff, 1946).

$$\mathbf{T}(x_i; \hat{\mathbf{n}}) = \lim_{\Delta S \rightarrow 0} \frac{\mathbf{F}}{\Delta S}. \quad (1.10)$$

If $\mathbf{T}^n$ is the stress vector acting at a point of surface to which $\hat{\mathbf{n}}$ is normal, then

$$\mathbf{T}^n = \sigma_{ij} n_j, \quad (1.11)$$

where σ_{ij} is a stress tensor and it represents the j^{th} component of the stress vector acting across a plane to which x_i -axis is normal.

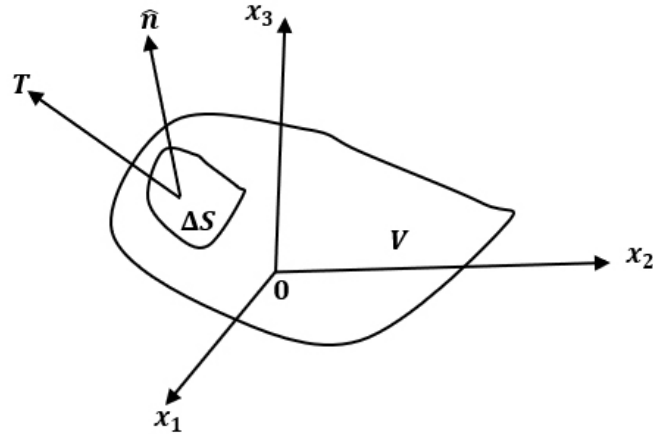


Figure 1.2: Stress vector

The deformation of the body accompanying stress is called *strain*. The deformation is called *dilatation* when the strain set up in the body is such that there is a change in volume of the body without change in its shape. But the deformation is called *shear* if the strain setup in the body is such that the shape of the body changes without any change in its volume. Thus, stress and strain occur simultaneously. The relationship between stress and strain for a deformable body is given by the generalized *Hooke's law* which states that stress and strain are linearly related. The tensor form of this law is given as

$$\sigma_{ij} = c_{ijkl}e_{kl}, \quad (i, j, k, l = 1, 2, 3) \quad (1.12)$$

where $c_{ijkl} = c_{jikl} = c_{ijlk}$ are the elastic constants and e_{kl} is strain. In general anisotropy, there are 21 elastic constants and these constants are 13, 9, 5 and 2 for monoclinic, orthotropic, transversely isotropic and isotropic materials respectively.

1.2.3 Equation of motion for elastic solid

Considered a continuous solid medium, every portion of which is contained within the volume V and bounded by a closed surface S . The resultant force acting on the matter within V (combine both forces, i.e., body force and surface force) must be equal to the inertia force. We shall calculate the x_i component of this force. Suppose

F be the body force per unit volume, T^ν be the surface force per unit area and $\rho(x_1, x_2, x_3)$ be the mean density of the medium. The component of the force of inertia acting on the mass contained within the volume element dV which is bounded by the surface element dS is $\rho \frac{\partial^2 u_i}{\partial t^2} dV$.

Then, we have

$$\int_V F_i dV + \int_S T_i^\nu dS = \int_V \rho \frac{\partial^2 u_i}{\partial t^2} dV. \quad (1.13)$$

If $\mathbf{T}^\nu$ is the stress vector acting at a point on a surface with unit normal vector $\boldsymbol{\nu}$, then the stress tensor can be expressed as:

$$T_i^\nu = \sigma_{ij} \nu_j, \quad (i, j = 1, 2, 3). \quad (1.14)$$

Making use of the relation of equation (1.14) and applying Gauss Divergence theorem, the second term in the left hand side of equation (1.13) becomes

$$\int_S \sigma_{ji} \nu_j dS = \int_V \frac{\partial \sigma_{ji}}{\partial x_j} dV. \quad (1.15)$$

Using Eq.(1.15) into (1.13), we get

$$\int_V (\sigma_{ji,j} + F_i) dV = \int_V \rho \frac{\partial^2 u_i}{\partial t^2} dV.$$

Since the region V of integration is arbitrary and the integrand of the equation (1.15) is continuous, it gives

$$\sigma_{ji,j} + F_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (i, j = x, y, z). \quad (1.16)$$

This is the equation of motion for a homogeneous elastic body in the presence of body forces.

1.3 Elastic waves

An *Elastic wave* is a motion in a medium in such a way that when particles are displaced, a force proportional to the displacement acts on the particles to restore

them to their original positions. When a material exhibits elasticity the ability to return to its original shape after deformation and its particles are disturbed into oscillatory motion, it generates and transmits an Elastic wave. For example, gases behave as elastic media: when compressed and then released, they return to their initial volume. Sound wave is a common example of elastic waves travelling through gases.

A *Seismic wave* is a type of vibration which is generated by earthquakes, explosions, or other powerful energy sources, and it travels through the Earth or along its surface. Earthquakes produce four main kinds of elastic waves: two types are called body waves: Primary and Secondary waves, which move through the Earth's interior, while the other two are known as surface waves: Love and Rayleigh waves, which travel along the Earth's outer layer. Seismographs are instruments that measure the strength (amplitude) and frequency of these waves, providing valuable insights into the Earth's internal structure. In addition, man-made seismic waves, generated during seismic surveys, are commonly used in the exploration of oil, natural gas, and for various engineering applications.

Among the two types of body waves, the primary wave or P -wave which travels the fastest, arriving at seismic stations before the secondary wave or S -waves. These P -waves are also known as longitudinal waves and move particles in the same direction in which the wave is travelling. Moreover, S -waves are also referred as shear or transverse waves and move particles in solid materials side to side, perpendicular to the direction in which the wave is moving. These waves can be polarized as SV and SH -waves.

Among the two types of surface waves, Love waves named after A.E.H. Love(1911), who first predicted their existence, travel faster. These waves occur when the solid layers near Earth's surface have different elastic characteristics with depth. The motion caused by Love waves is entirely horizontal and occurs perpendicular to the wave's direction of travel, with no movement in the vertical or forward-backward

(longitudinal) directions. Since surface waves, including Love waves, spread out in two dimensions from the source rather than in three, their energy remains more concentrated. This makes them produce strong signals at seismic stations, even when generated by earthquakes far away. The second main type of surface wave is the Rayleigh wave, named after British physicist Lord Rayleigh(1885), who was the first to mathematically prove their existence. These waves move along the outer surface of elastic solids like the Earth. Their motion is a combination of longitudinal compression and dilation that results in an elliptical motion of points on the surface. Of all seismic waves, Rayleigh waves spread out most in time, producing a long wave duration on seismographs.

1.4 Reflection and refraction at a plane interface

In plane motions in an isotropic medium, the system of incident, reflected and refracted waves satisfy the conditions of continuity of the stresses and the displacements at the interface of the two media. Suppose two half-spaces, $M : \{(x_1, x_2); -\infty < x_2 \leq 0\}$ and $M' : \{(x_1, x_2); 0 \leq x_2 < \infty\}$ are separated by $x_2 = 0$. If a wave either P or SV in the half-space, M is incident at the plane interface, there are two each reflected and refracted waves. If the incident wave is SH -wave, there are one each reflected and refracted SH -wave. The unit propagation vectors of the system of incident, reflected and refracted waves are shown in Figure 1.3.

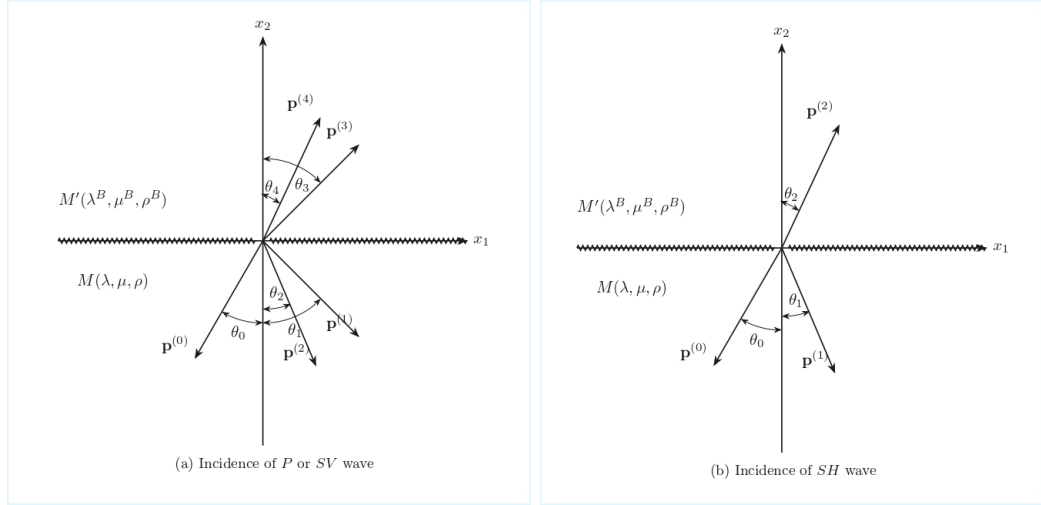


Figure 1.3: Incident, reflected and refracted waves

The material properties of the half-space, M carrying the incident and the reflected waves are given by the Lamé elastic constants (λ & μ) and density, ρ while those of half-space, M' are given by λ^B , μ^B and ρ^B .

The incident, the reflected and refracted waves are represented by

$$\mathbf{u}^{(n)} = A_n \mathbf{d}^{(n)} \exp(i\eta_n), \quad n = 1, 2, 3, 4 \quad (1.17)$$

where A_n is amplitude constant, $\mathbf{d}^{(n)}$ is the direction of propagation of the particles and the expression of η_n with k_n as wavenumber is given by

$$\eta_n = k_n(x_1 p_1^{(n)} + x_2 p_2^{(n)} - c_n t).$$

The index $n = 0$ represents for the incident wave as

$$\mathbf{p}^{(0)} = \sin \theta_0 \boldsymbol{\epsilon}_1 + \cos \theta_0 \boldsymbol{\epsilon}_2,$$

where $\mathbf{d}^{(0)} = \mathbf{p}^{(0)}$ & $c_0 = c_L$ for the incident longitudinal wave and $\mathbf{d}^{(0)} \cdot \mathbf{p}^{(0)} = 0$ & $c_0 = c_T$ for the incident transverse wave.

The two types of transverse waves, SV and SH -waves are distinguished as $\mathbf{d}^{(0)} = \mathbf{i}_3 \times \mathbf{p}^{(0)}$ & $c_0 = c_T$ for incident SV -wave and $\mathbf{d}^{(0)} = \mathbf{i}_3$ & $c_0 = c_T$ for the incident SH -wave.

The reflected longitudinal and transverse waves are labelled by $n = 1$ and $n = 2$ respectively, while the refracted longitudinal and transverse waves are labelled by $n = 3$ and $n = 4$, respectively. These waves are given by

$$\begin{aligned}\mathbf{p}^{(1)} &= \sin \theta_1 \mathbf{i}_1 - \cos \theta_1 \mathbf{i}_2, & \mathbf{d}^{(1)} &= \mathbf{p}^{(1)}, & c_1 &= c_T, \\ \mathbf{p}^{(2)} &= \sin \theta_2 \mathbf{i}_1 - \cos \theta_2 \mathbf{i}_2, & \mathbf{d}^{(2)} &= i_3 \times \mathbf{p}^{(1)}, & c_2 &= c_T, \\ \mathbf{p}^{(3)} &= \sin \theta_3 \mathbf{i}_1 + \cos \theta_3 \mathbf{i}_2, & \mathbf{d}^{(3)} &= \mathbf{p}^{(3)}, & c_3 &= c_L^B, \\ \mathbf{p}^{(4)} &= \sin \theta_4 \mathbf{i}_1 + \cos \theta_4 \mathbf{i}_2, & \mathbf{d}^{(4)} &= i_3 \times \mathbf{p}^{(4)}, & c_4 &= c_T^B.\end{aligned}$$

The reflected and refracted SH -waves are respectively given by

$$\begin{aligned}\mathbf{p}^{(1)} &= \sin \theta_1 \mathbf{i}_1 - \cos \theta_1 \mathbf{i}_2, & \mathbf{d}^{(1)} &= \mathbf{i}_3, & c_1 &= c_T \text{ \& } \\ \mathbf{p}^{(2)} &= \sin \theta_2 \mathbf{i}_1 + \cos \theta_2 \mathbf{i}_2, & \mathbf{d}^{(2)} &= \mathbf{i}_3, & c_2 &= c_T^B.\end{aligned}$$

The Snell's law which gives the relation among the angles of incident, reflected and refracted waves with phase velocities is given by

$$\sin \theta_0 / c_0 = \sin \theta_i / c_i, \quad (i = 1, 2, 3, 4).$$

1.4.1 Harmonic Waves

Let us consider an expression for the longitudinal displacement of the form as (Achenbach 1976)

$$u(x, t) = A \cos(x - ct), \tag{1.18}$$

where the amplitude A is independent of x and t .

Eq.(1.18) is of the general form $f(x - ct)$ and thus clearly represents a travelling wave. The argument $k(x - ct)$ is called the phase of the wave and the points of constant phase are propagated with the phase velocity, c . At any instant t , $u(x, t)$ is a periodic function of x with wavelength λ which is represented by $\lambda = \frac{2\pi}{k}$. The quantity, $k = \frac{2\pi}{\lambda}$ is termed the wavenumber. At any position, the displacement $u(x, t)$ is time-harmonic with time period T and $T = 2\pi/\omega$. The circular frequency, ω is

given by

$$\omega = kc. \quad (1.19)$$

It follows that an alternative representation of $u(x, t)$ as

$$u(x, t) = A \cos[\omega(\frac{x}{c} - t)]. \quad (1.20)$$

Eqs. (1.18) and (1.20) represent the equations of harmonic waves.

1.5 Thermoelasticity

Thermoelasticity is the study of effect of change of temperature on the mechanical behaviour of the elastic materials and vice versa. In other words, the coupled phenomena of the interplay between thermal and mechanical fields are discussed. In classical thermoelasticity theory, an instantaneous transfer of heat is considered which is not physically accurate for all situations, especially those involving rapid temperature changes like thermal shocks. In generalized thermoelasticity, the limitation in classical is incorporated with more complex heat transfer model that account for finite speed of heat propagation. Application of such materials are in numerous fields like acoustic, aeronautics, chemical, and nuclear engineering. Biot(1956) developed the classical theory of coupled thermoelasticity and obtained the coupling equations of elasticity and heat conduction. This theory introduced the concept of thermal force as a generalized force using the principle of virtual work involving the product of the temperature and a virtual entropy displacement. The theory also gives the thermoelastic potential by a single invariant of the elastic and thermostatic properties of the medium. The weakness in this theory is that the parabolic heat equation indicates an infinite speed for thermal signal and thermal disturbance still has an effect when it is infinitely far away from the heat source.

Let V stand for any material volume that is enclosed by the surface A . The law

of conservation of energy for V can be expressed as

$$\frac{d}{dt} \int_V \left[\frac{1}{2} \dot{u}_i \dot{u}_i + U \right] \rho dV = \int_V [\rho F_i \dot{u}_i + \rho S_1] dV + \int_A [\tau_{ij} \dot{u}_i - q_j] \nu_i dA,$$

where q_i is the heat flux vector, U is the internal energy per unit mass, u_i is the displacement component, ρ is the mass density assumed to be independent of time t , F_i is the external force component per unit mass, S_1 is the heat source density, ν_i is the unit outward normal vector to the surface A and the symbol "." corresponds to the time derivative.

Lord and Shulman(1967) introduced generalized thermoelastic model using modified Fourier's law of heat conduction and introduced the concept of a relaxation time. The time required to accelerate the heat flow corresponds to this relaxation period. The modified version of the heat conduction law is

$$-K_{ij} T_{,i} = q_i + \tau_0 \dot{q}_i,$$

where K_{ij} and τ_0 represent the coefficient of heat conduction and relaxation time respectively. Green and Lindsay(1972) developed generalized thermoelasticity theory by introducing temperature rate-dependent in the consecutive variables with two relaxation times. Green and Naghdi(1991, 1992, 1993) developed three thermomechanical theories for deformable continua that rely on an entropy balance law rather than an entropy inequality and labeled as type I, II and III. The type I thermoelasticity coincides with the classical theory of thermoelasticity. In the type II, heat is allowed to flow via thermal waves but without releasing energy and as a result, it is also known as thermoelasticity without energy dissipation. The heat equation of type III, where the heat flux is a combination of types I and II, includes both types I and II as limiting cases. Furthermore, the type III thermoelasticity permits the constitutive functions for free energy, stress, entropy and heat flux to depend on the strain, time derivative of thermal displacement, thermal displacement gradient and time derivative of thermal displacement gradient. The heat flux is partially

determined by the Helmholtz free energy potential although the dissipation of energy is permitted. More about the thermoelasticity, one may explore from Boley and Weiner(1960), Green and Adkins(1960), Day(1985) and Iesan and Scalia(1996).

1.6 Maxwell's Equation

Maxwell's equations are a set of coupled partial differential equations that, together with the Lorentz force law form the foundation of classical electromagnetism, classical optics, electric and magnetic circuits. The equations provide a mathematical model for electrical, optical, and radio technologies, such as power generation, electric motors, wireless communication, lenses, radar etc. They describe how electric and magnetic fields are generated by charges, currents, and changes of the fields. The equations are named after the physicist and mathematician, James Clerk Maxwell(1861; 1862). In partial differential equation form, Maxwell's equations can be written as Gauss's law, Gauss's law for magnetism, Faraday's law, and Ampere-Maxwell law.

The Maxwell's equations corresponding to linear electrodynamics for a homogeneous, thermally and electrically conducting elastic solid are given by (Ezzat and Ellal, 2009)

$$\text{curl } \mathbf{h} = \mathbf{J} + \dot{\mathbf{D}}, \quad \text{curl } \mathbf{E} = -\dot{\mathbf{B}}, \quad \text{div } \mathbf{B} = 0, \quad (1.21)$$

$$\text{div } \mathbf{D} = \rho_e, \quad \mathbf{B} = \mu_0 \mathbf{H}, \quad \mathbf{D} = \epsilon_0 \mathbf{E},$$

where $\mathbf{H}$ is magnetic intensity, $\mathbf{h}$ is induced magnetic field, $\mathbf{J}$ is current density vector, $\mathbf{D}$ is electric displacement, $\mathbf{E}$ is electric field, $\mathbf{B}$ is magnetic induction, ρ_e is the current density, ϵ_0 is electric permeability and μ_0 is magnetic permeability.

1.7 Modified Ohm's law

Ohm's law states that the electric current through a conductor between two points is directly proportional to the potential difference across the points. Mathematically,

this relation is defined as

$$V = RI, \quad (1.22)$$

where I is the current through the conductor, V is the voltage measured across the conductor and R is the resistance of the conductor. More specifically, Ohm's law states that the R in this relation is constant, independent of the current. This law is named after the German physicist Georg Ohm(1827) which described the measurements of applied voltage and current through simple electrical circuits containing various lengths of wire.

In physical science, the vector form of the Ohm's law in electromagnetic materials is represented by

$$\mathbf{J} = \sigma_0 \mathbf{E} \quad (1.23)$$

where σ_0 is the conductivity of the material.

The modified Ohm's law for finite conductivity in generalized magneto thermoelastic medium is given by (Ezzat and Ellal, 2009)

$$\mathbf{J} = \sigma_0 [\mathbf{E} + \mu_0 \dot{\mathbf{u}} \times \mathbf{H}] - \kappa_0 \nabla T, \quad (1.24)$$

$$\vec{\mathbf{J}} = \sigma_0 [\mathbf{E} + (\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{B})], \quad (1.25)$$

where $\mathbf{u}$ is displacement and T is reference temperature.

The modified Ohm's law for finite conductivity in generalized magneto micropolar thermoelastic medium is given by (Ezzat and Awad, 2009)

$$\mathbf{J} = \sigma_0 \{ \mathbf{E} + \mu_0 \dot{\mathbf{u}} \times \mathbf{H}_0 \} + \rho_e \dot{\mathbf{u}} - \kappa_0 \nabla T, \quad (1.26)$$

where κ_0 is the coefficient connecting the temperature gradient and the electric current density, ρ_e .

1.8 Constitutive relations

A constitutive equation of a continua is a relation between two or more physical quantities that is specific to a material or substance or field, and approximates its response to external stimuli, usually as applied fields or forces. This equation with other equations governing physical laws are used to solve physical problems related with vibration and deformation of the materials. In simple isotropic materials, it is defined by Hooke's law as

$$\tau_{ij} = \lambda e_{kk} \sigma_{ij} + 2\mu e_{ij}, \quad 2e_{ij} = u_{i,j} + u_{j,i}. \quad (1.27)$$

A magneto thermoelastic material is a material that exhibits a coupled interaction between magnetic, thermal, and elastic properties. This means that changes in temperature can induce magnetic fields and stresses, and conversely, magnetic fields and stresses can affect temperature distribution within the material. The constitutive relations for the generalized magneto-thermoelastic continua are represented as (Ezzat and Ellal, 2009; Zorammuana and Singh, 2022)

$$\sigma_{ij} = 2\mu e_{ij} + \lambda \Sigma \delta_{ij} - \gamma \delta_{ij} T, \quad (1.28)$$

$$\tau_{ij} = \mu_0 (H_i h_j + H_j h_i - H_k h_k \delta_{ij}), \quad (i, j, k = 1, 2, 3) \quad (1.29)$$

where T is increase in the reference temperature, τ_{ij} is Maxwell's tensor and γ is thermal parameter.

A micro-polar magneto thermoelastic material refers to a material that exhibits coupled behaviour of mechanical, thermal, and magnetic fields, taking into account micro-rotations and couple stresses. The constitutive equations for the linear theory of micropolar generalized magneto-thermoelastic materials are represented by (Ezzat and Awad, 2009)

$$\sigma_{ij} = \lambda u_{k,k} \delta_{ij} + (\mu + \alpha) u_{j,i} + (\mu - \alpha) u_{i,j} - 2\alpha e_{ijk} w_k - \gamma T \delta_{ij}, \quad (1.30)$$

$$\mu_{ij} = \epsilon w_{k,k} \sigma_{ij} + (\nu + \beta) w_{j,i} + (\nu - \beta) w_{i,j}, \quad (1.31)$$

where α , ϵ , ν and β are micropolar parameters, μ_{ij} is couple stress tensor and w is micro-rotation.

The equation of energy in the absence of heat source is given by

$$\rho T_0 \dot{\eta} = -q_{i,i}. \quad (1.32)$$

The entropy, η in an isotropic media may be written in terms of temperature as

$$\rho \eta = \frac{\rho C_e}{T_0} T + \gamma \nabla \cdot \mathbf{u}, \quad (1.33)$$

where C_e is specific heat.

The generalized Fourier's law including the current density effect in micropolar generalized magneto thermoelastic material is given by (Ezzat and Awad, 2009)

$$q_i + \tau_0 \dot{q}_i = -k T_{,i} + \pi_0 J_i, \quad (1.34)$$

where J_i is component of current density.

1.9 Scope of the study

Wave propagation and phenomena of reflection and refraction from a boundary surface is an interesting subject that deals with numerous problems in various fields, i.e Seismology, geophysics, Earthquake engineering, tele-communication, medicines and signal processing. These waves are useful in detection of notches and faults in different types of materials such as in railway tracks, buried land-mines, etc. Seismic waves are useful tool for investigating the internal structure of earth and also used for exploration of valuable materials such as minerals, crystals, fluids (oils, water), etc.

We observe the practical application of elastic waves in many activities from our daily life from communication to medical technology. These applications utilize various types of waves like electromagnetic, acoustic, and seismic, each with unique properties that make them suitable for specific tasks. In essence, understanding

wave propagation is crucial for comprehending a wide range of natural phenomena and for developing various technologies that benefit society in numerous ways. Sky wave propagation enables long-distance radio transmission by bouncing signals off the ionosphere. This phenomenon is essential in communication systems, where electromagnetic waves carry information over long distances, such as in radio, television, and mobile networks. In medicine, wave propagation enables techniques like ultrasound imaging, which uses high-frequency sound waves to visualize internal organs. Seismology relies on the propagation of seismic waves through the Earth to study earthquakes and underground structures. Acoustic applications, such as sound systems and noise-cancelling headphones, also depend on the behaviour of sound waves in air. Additionally, technologies like radar and sonar use wave propagation to detect objects and measure distances by analyzing reflected waves. Whether in air, water, or solid materials, wave propagation plays a critical role in transmitting signals, energy, and information across various fields.

1.10 Review of Literature

Green and Laws(1972) present an alternate generalized thermoelastic theory. These theories have been established using modifying Fourier's heat conduction equation or correcting the energy equation and Neuman-Duhamel relation(Chandrasekhariaah and Srinath, 1998). Nowacki(1975) discussed different problems of thermoelasticity and gave the Duhamel-Newmann relations for an anisotropic body. Anwar and Sherief(1988) adopted the state space approach for the solution of coupled thermoelastic problems for the solution of one-dimensional problem in generalized thermoelasticity. Sherief(1993) studied the equations of generalized thermoelasticity with one relaxation time for $1D$ problem including heat sources are cast into math form using the state space and Laplace transform techniques. Youssef(2011) constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without

energy dissipation. Several authors, such as Neyfeh and Nasser(1971), Paria(1966), Puri(1973) and Hetnarski and Ignazick(1999) contributed in the development of generalized thermoelastic theory.

McCarthy(1972) obtained two purely mechanical transverse and two longitudinal waves in generalized theromoelastic continuum. He established the governing principles for how acceleration waves of arbitrary configuration and strength propagate. Agarwal(1978) studied the surface waves in generalized thermoelasticity. Abd-Alla(1999) examined how the initial stress and gravity field affect the creation of waves in an orthotropic elastic solid medium. Tomar and Singh(2006) found the frequency equations for Stoneley waves at unbonded and bonded interfaces between two dissimilar microstretch elastic half-spaces. Abouelregal(2011) studied the Rayleigh waves in a thermoelastic homogeneous isotropic solid half-space in the context of dual-phase-lag model. The analytical expressions for the displacement, temperature, and thermal stress amplitudes as well as the equation of phase velocity of the Rayleigh wave have been derived. Singh and Pal(2011) obtained the frequency equation of the surface wave in the generalized thermoelastic material with voids. Abo-Daheb(2014) studied the problem of surface waves in a generalized magneto-thermoelastic materials with voids and initial stress using Green-Lindsay model and obtained the frequency equation of the surface waves in the medium. Abo-Daheb(2015) investigated the propagation of Stoneley waves in magneto-thermoelastic materials with voids and two thermal relaxation times in the context of a Green-Lindsay model and obtained the frequency equation of the Stoneley waves in the medium. Goyal et al.(2016) used Eringen's theory of swelling porous media to derive the frequency equation of Rayleigh-type waves. The non-dimensional frequency of surface wave propagation was determined by (Farhan and Abd-Alla, 2018) as the result of solving an algebraic equation containing a determinant whose elements include the rotation parameter, the material parameters, and the direction of the applied magnetic field. Kumar et al.(2018) found the dispersion equation of Stoneley waves in

the generalized thermoelastic material with diffusion. Singh and Tochhawng(2019) dealt the problem of the propagation of surface waves, particularly Stoneley and Rayleigh waves in thermoelastic materials with voids and the frequency equations of the Stoneley waves at the bonded and unbonded interfaces are obtained. Chandavath et al.(2023) studied the wave propagation along the interface of poroelastic cylindrical solid and frequency equation of stoneley waves are obtained. Debanath and Singh(2025) studied the nature of human tissue under the context of dual phase lag thermoelastic diffusion theory using the principle of propagation of Rayleigh wave and irrational secular equations of Rayleigh waves are derived. Wave propagation in elastic substances has been studied for a very long time. The early investigators in the problem of wave propagation can be seen from Love(1944), Rayleigh(1945), Ewing et al.(1957), Brekhovskikh(1960), Lindsay(1960), Cagniard(1962), Bullen(1963), Kolsky(1963), Biot(1965), Miklowitz(1966), Tolstoy and Clay(1966), Morse and Ingard(1968), Payton(1983), Hanyga(1985), Graff(1991), Nayfeh(1995), Udias(1999), Chapman(2004), Singh and Tomar(2007), Sato and Fehler(2009), Barber(2010), Hetnarski(2014) and Altenbach and Öchsner(2020).

Borcherdt(1982) studied the problem of reflection and refraction of general P and S wave in elastic and anelastic solids. He discussed reflection and refractions, physical behaviour of the waves, and the characteristics of critical angles. Sinha and Elsibai(1996) investigated the reflection of thermoelastic waves at a homogeneous, isotropic, and thermally conducting elastic solid half-space. He obtained the expressions for the partition of the energy for various values of the angle of incidence. Sinha and Elsibai(1997) obtained the analytical expressions for the partition of energy for the reflected and refracted waves at an interface between two semi-infinite media. Sharma et al.(2003) investigated the problem of reflection of thermoelastic wave for the insulated and isothermal condition in the stress free boundary and obtained the reflection coefficients of the various reflected waves. Chakraborty and Singh(2011) derived the amplitude ratios of the reflected and refracted ther-

moelastic waves at the plane interface between two dissimilar thermoelastic solid half-spaces. Wei et al.(2016) discussed the reflection and refraction phenomenon due to an obliquely incident longitudinal wave at a plane interface between thermoelastic and porous thermoelastic medium. They obtained the expressions of amplitude ratios of various reflected and refracted waves. Many problems of reflection and refraction of elastic waves are in open literature, i.e., by Parfitt and Eringen (1969), Sinha and Sinha(1974), Tomar and Gogna (1995), Singh and Kumar(1998a, 1998b), Singh (2000), Pujol(2003), Kaur and Tomar(2004), Singh and Singh(2004), Tomar and Arora(2006), Tomar and Khurana(2008), Markov(2009), Kumar and Saini(2012), Goyal and Tomar(2015), Kumar and Saini(2016), Shekhar and Parvez(2016), Zoram-muana and Singh(2017), Kumar et al.(2019), Markov et al.(2019), Zhou et al.(2019), Liannngenga and Singh(2020), Wang et al.(2020) and Hilal et al.(2021).

Dhaiwal and Sherief(1980) derived the variation principle and uniqueness theorem for the generalized anisotropic thermoelastic material. Chaudhuri and Deb-nath(1983) investigated the propagation of plane harmonic waves in an infinite conducting thermo-elastic solid permeated by a primary uniform magnetic field when the entire elastic medium is rotating with a uniform angular velocity. Sharma and Chand(1998) discussed the problem of transient magneto thermoelastic waves using potential function based on Lord and Shulman thermoelastic theory. Sherief and Helmy(2002) used the techniques of Laplace and exponential Fourier transform to the distributions of the temperature, the displacement, the stress and the induced magnetic and electric fields. Wang and Dai(2004) introduced a theoretical method to analyse the magnetothermoelastic responses and perturbation of the magnetic field in the orthotropic thermoelastic cylinder subjected to thermal shock. Youssef(2010) proposed a new thermoelasticity theory considering the heat conduction equation with fractional order and proved the uniqueness theorem. Othman and Abbas(2012) studied the solution of a thermal-shock problem of generalized non-homogeneous thermoelastic isotropic hollow cylinder using finite-element method. Sarkar and Lahiri(2012)

used the normal mode analysis and eigenvalue approach techniques to obtain the kinematic variables in the three dimensional thermoelastic problem without energy dissipation.

Cosserat and Cosserat(1909) made significant advances in the Mathematical characterization of solid bodies with microstructure. Mindlin and Tiersten(1962) proposed a couple stress theory in which the rotation of a material point equal to the local rotation of the surrounding medium. Mindlin(1964) introduced a linear theory of three dimensional elastic continuum incorporating the idea of unit cell and discussed some properties of crystal lattice. Tauchert et al.(1968) developed the linear theory of micropolar thermoelasticity. A generalized theory of linear micropolar thermoelasticity that admits the possibility of “second sound” effects was established by Boschi and Iesan(1973). Dost and Tabarrok(1978) presented the generalized micropolar thermoelasticity by using Green–Lindsay theory. This theory has been extended to the theory of microstretch continuum(Eringen, 1990), where the material points of these solids can stretch and contract independently of their translations and rotations. This continuum model explains those materials of Bravais lattice with basis and can model composite materials reinforced with chopped fibers and porous solids filled with gas or inviscid fluids. Iesan and Pompei(1995) developed the static theory of microstretch elastic solids and included a counterpart of the Boussinesq Somigliana-Galerkin solution to the problem of an infinite solid subject to concentrated body loads. Cialletta(1999) established a theory of micropolar thermoelasticity without energy dissipation that permits propagation of thermal waves at a finite speed. Neff et al.(2009) developed a mean field modeling of random isotropic cauchy material that leads to the well-established micro-stretch model. Othman et al.(2019) studied the deformation of an infinite microstretch generalized thermoelastic medium rotating with some angular frequency.

Othman and Singh(2007) studied a two-dimensional problem in a micropolar thermoelastic medium and the entire elastic medium is rotating with a uniform angular

velocity. The formulation is implemented under five theories of the generalized thermoelasticity: *LS* theory, *GL* theory, *GN – II* theory and *CT* theory, as well as the coupled theory. Using normal mode analysis, they obtained exact expressions for the relevant variable. Moncef(2009) studied the variational principle for the governing equations and found the reciprocity theorem. Aliawalia et al.(2016) investigated a two-dimensional problem and derived analytical expressions for the displacement components, normal stress, temperature distribution, and microelongation. Kumar et al.(2016) presented the explicit expressions for the fundamental solution of the system of equations in the case of steady vibrations. Dhabu and Raghukanth(2019) derived the explicit solutions for displacements and rotations and obtained the solutions for displacements. Nesarhosseini et al.(2023) studied the free vibrations of beam-type structures subjected to rapid heating and the solution in time domain is done by the Newmark algorithm based on the constant average acceleration technique. Some problems related with micropolar thermoelastic medium may be mentioned as Nowacki(1970), Shanker and Dhaliwal(1975), Chirita(1979), Passarella(1996), Kumar and Deswall(2001), Kumar and Aliawalia(2007), Sargsyan(2012), Zakaria(2012), Kumar and Abbas(2013), Ezzat et al(2014), Othman and Mondal(2020), Yadav(2021), Kumar et al.(2022), Kundu et al.(2023), Abouelregal et al.(2023) and Kumar and Nazir(2023).

Knopoff(1955) examined the interaction between elastic motion and magnetic field by presenting the field equation and constitutive relation. Nowacki(1971) examined a two-dimensional problem in the linear theory of micropolar generalized magneto-thermoelasticity. Nath et al.(1998) dealt with the problem of magneto-thermoelastic surface waves in a micropolar medium and they derived the general frequency equations. El-Karamany and Ezzat(2004) discussed the analytical aspects of the formation of boundary integral equation for generalized linear micropolar thermoelasticity. Sherief et al.(2005) resolved an axisymmetric thermal shock issue and demonstrated the uniqueness theorem for generalized micropolar thermoelastic materials. Khan et

al.(2005) proposed a model of the propagation of harmonic plane waves through a non-local micropolar medium with temperature-dependent and discussed the influence of non-locality and rotation effects on the reflected waves. The response of micropolar thermoelastic medium with voids due to various sources under Green-Naghdi theory was established by Othman and Atwa(2012). Sarkar(2014) investigated the coupled problem of two dimensional half-space solid using modified Ohm's law with eigenvalue approach techniques. Said et al.(2017) examined the effects of rotation and magnetic field on the total deformation of a micropolar thermoelastic medium with an internal heat source using normal mode analysis.

The subject of wave propagation and their phenomena of reflection and transmission from boundary surface and interface through materials modeled using classical continuum and microcontinuum theories are very interesting area of research. Chattopadhyay and Choudhury(1990) studied of propagation reflection and transmission of magnetoelastic shear waves in a self-reinforced medium. Tomar and Gogna(1992) investigated the problem of a longitudinal microrotational wave impinging at a plane discontinuity between two micropolar elastic solids. They obtained analytically and numerically the amplitude ratios of the reflected and refracted waves. Chattopadhyay et al.(2002) studied the propagation of plane waves in fibre-reinforced media where the expressions of phase velocities, reflection coefficients of *quasi* – *P* (*qP*) and *quasi* – *SV* (*qSV*) are obtained in terms of propagation vectors. Chaudhary et al.(2004) studied the reflection and transmission of plane *SH*-wave through a self-reinforced elastic layer and they found the expressions for reflection and transmission coefficients. Othman and Song(2008) studied the reflection of plane waves from an isotropic magneto-thermoelastic rotating half space and obtained the reflection coefficients of the reflected waves. Othman and Kumar(2009) presented the model equations of generalized magneto-thermoelasticity in an isotropic perfectly conducting elastic medium. Yu et al.(2013) discussed the problem of fractional order generalized electromagneto-thermo-elasticity in welded contact and introduced a unified

framework for various thermoelastic models with electromagnetic effects. Abd-Alla et al.(2016) investigated the reflection of plane harmonic waves from a semi-infinite elastic medium in the presence of a magnetic field and derived the corresponding reflection coefficient. Singh(2017) studied a problem on the reflection of elastic waves at a plane surface of an elastic half-space and the reflection coefficients of reflected P and SV are obtained. Abo-Daheb(2018) solved the governing equation of generalized magneto thermoelastic materials using Lamé's potentials method in the context of classical dynamical and Lord-Shulman theories.

Lata(2018) considered a layered medium of two semi-infinite nonlocal elastic solids with intermediate transversely isotropic magnetothermoelastic solid and the amplitude ratios of various reflected and refracted waves are obtained by using appropriate boundary conditions. Perati et al.(2019) studied the reflection and transmission phenomena in poroelastic semi-infinite solid cylinder and found the reflection and transmission coefficients. Ala et al.(2020) discussed the propagation of shear waves between self-reinforced poroelastic medium and poroelastic half space and computed the phase velocity. Zorammuana and Singh(2022) introduced the magnetic and rotational effects on the propagation of waves in transversely isotropic fiber-reinforced thermoelastic material with the help of Lord Shulman theory. Kumar and Tomar(2022) investigated the effect of induced electric field due to applied magnetic field on the reflection and transmission coefficients of the elastic waves in the half-spaces of magneto-elastic materials with voids. Gurijala et al.(2023) explored the reflection and transmission phenomena in axially symmetric waves in two half-spaces, in which the lower one is poroelastic half-space, and the upper one is self-reinforced half space and they derived reflection and transmission coefficients. Zorammuana and Singh(2024) studied the problem of reflection and transmission of inhomogeneous plane waves caused by incident (qP) and (qSV) waves at the interface between a fiber-reinforced thermoelastic solid under magnetic field and a fiber-reinforced thermoelastic solid is examined mathematically using Lord-Shulman theory and they

obtain the expressions of amplitude ratios and energy ratios numerically. The problems which are related with wave propagation in various thermoelastic materials have been studied since long and they are in open literature, i.e., Ignaczack(1979), Singh and Kumar(1998), Kumar (2000), Ezzat et al.(2001), Kumar and Deswal(2002), Abd-Alla et al.(2003), Ezzat(2004), Othman(2004), Song et al.(2004), Youssef(2006), Othman and Song(2006), Dai and Wang(2006), Tomar and Singh(2006a), Tomar and Singh(2006b), Abbas and Youssef(2009), Singh(2009), Othman and Lotfy(2009), Abbas et al.(2011), Abd-Alla et al.(2012), Singh(2013), Abd-Alla et al.(2014), Singh et al.(2014), Abbas(2015), Kumar et al.(2018), Abo-Daheb and Lotfy(2015), Abd-Alla and Asker(2016), Abd-Alla et al.(2016), Zorammuaana and Singh(2016), Roy et al.(2017), Deswal et al.(2019), Abd-Alla et al.(2019), Abo-Daheb et al.(2020), Alharbi et al.(2020), Lotfy et al.(2020a), Lotfy et al.(2020b), Kalkat et al.(2020), Mahdy et al.(2020), Sarkar and De(2020), Lotfy et al.(2021), Ismail et al.(2022), Kumar et al.(2022), Lotfy and El-Bary (2022) and Singh et al.(2022).

Chapter 2

Elastic wave in half-space of electro magneto thermo elastic solid¹

The subject of wave and vibrations in different elastic continuum has been a matter of concern for many researchers since long. It is used in various field like in the exploration of mines and petroleum, earthquake engineering, Seismology, Geophysics, etc. More specific, wave phenomena in thermoelastic materials has wide applications in engineering. Othman and Song(2008) studied the reflection of plane waves from an isotropic magneto-thermoelastic rotating half space and obtained the reflection coefficients of the reflected waves. Lotfy et al.(2019) discussed the problem of the magnetic and rotational effects on a fiber-reinforced thermoelastic solid based on two coupled thermoelastic theory with gravitational effect using normal mode analysis.

Symbols	Meaning	Symbols	Meaning
t	time	ϵ_1	$\frac{\gamma^2 T_0}{\rho C_e (\lambda + 2\mu)}$
β_1	$\frac{\sigma_0 \mu_0}{\eta}$	η	$\frac{\rho C_e}{\kappa}$
c_1^2	$\frac{\lambda + 2\mu}{\rho}$	c^2	$\frac{1}{\epsilon_0 \mu_0}$
ϵ_0	electric permeability	γ	$(3\lambda + 2\mu)\alpha_T$
α	$\frac{\mu_0 H_0^2}{\lambda + 2\mu}$	ϵ_2	$\frac{c_1^2}{c^2}$
β^2	$\frac{\lambda + 2\mu}{\mu}$	c_2^2	$\frac{\mu}{\rho}$
θ_i	reflected angle (i = 1,2,3)	α_T	linear thermal expansion

Table 2.1: Nomenclature

¹Singh, S.S. and Boruah S.(2023). "Effect of Magnetic and Thermal Parameters on the Reflected Waves in Generalized Magneto Thermoelastic Materials", *Mat Plus*, **2**, 33-45.

In the present work, we are concerned with the magnetic and thermal effects on the reflected waves in generalized magneto thermoelastic materials. The phase velocities corresponding to longitudinal, transverse and thermal waves are obtained. The amplitude and energy ratios are derived analytically and computed numerically. We verify our results through the conservation law of energy.

2.1 Governing Equations

The equation of motion in the absence of body force and heat source for linearly homogeneous magneto thermoelastic materials are

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i, \quad (2.1)$$

where $F_i = \mu_0(\mathbf{J} \times \mathbf{H})_i$.

The heat conduction equation for such a medium is

$$\kappa T_{,ii} = \rho C_e(T_{,t} + \tau_0 T_{,tt}) + \gamma T_0(\Sigma_{,t} + \tau_0 \Sigma_{,tt}), \quad (2.2)$$

where $\Sigma = \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}$.

We consider a two dimensional problem in xy -plane of the Cartesian system so that $u_i = (u_1, u_2, 0)$ and $\frac{\partial}{\partial z} \equiv 0$. The stress tensors are given by

$$\begin{aligned} \sigma_{11} &= (\lambda + 2\mu)u_{1,x} + \lambda u_{2,y} - \gamma T, & \sigma_{22} &= (\lambda + 2\mu)u_{2,y} + \lambda u_{1,x} - \gamma T, \\ \sigma_{12} &= \mu(u_{1,y} + u_{2,x}). \end{aligned} \quad (2.3)$$

The components of magnetic intensity vector in the medium are represented by

$$H_1 = 0, \quad H_2 = 0, \quad H_3 = H_0 + h(x, y, t).$$

The current density $\mathbf{J}$ is parallel to induced electric field $\mathbf{E}$. Thus, the linearized components of $\mathbf{J}$ after using (1.24) are given by

$$J_1 = \sigma_0(E_1 + \mu_0 H_0 \frac{\partial u_2}{\partial t}) - \kappa_0 \frac{\partial T}{\partial x}, \quad (2.4)$$

$$J_2 = \sigma_0(E_2 - \mu_0 H_0 \frac{\partial u_1}{\partial t}) - \kappa_0 \frac{\partial T}{\partial y}, \quad J_3 = 0. \quad (2.5)$$

Equations (1.21) and (1.24) give the following three equations

$$\frac{\partial h}{\partial y} = \sigma_0(E_1 + \mu_0 H_0 \frac{\partial u_2}{\partial t}) - \kappa_0 \frac{\partial T}{\partial x} + \epsilon_0 \frac{\partial E_1}{\partial t}, \quad (2.6)$$

$$\frac{\partial h}{\partial x} = -\sigma_0(E_2 - \mu_0 H_0 \frac{\partial u_1}{\partial t}) + \kappa_0 \frac{\partial T}{\partial y} - \epsilon_0 \frac{\partial E_2}{\partial t}, \quad (2.7)$$

$$\frac{\partial E_1}{\partial y} - \frac{\partial E_2}{\partial x} = \mu_0 \frac{\partial h}{\partial t}. \quad (2.8)$$

Using Equations (2.4) and (2.5), we get

$$\begin{aligned} F_1 &= \sigma_0 \mu_0 H_0 (E_2 - \mu_0 H_0 \frac{\partial u_1}{\partial t}) - \kappa_0 H_0 \mu_0 \frac{\partial T}{\partial y}, \\ F_2 &= -\sigma_0 \mu_0 H_0 (E_1 + \mu_0 H_0 \frac{\partial u_2}{\partial t}) + \kappa_0 H_0 \mu_0 \frac{\partial T}{\partial x}, \quad F_3 = 0. \end{aligned} \quad (2.9)$$

Putting the values of σ_{ij} and F_i in (2.3) and (2.9) into (2.1), we get

$$(\lambda + \mu) \frac{\partial \Sigma}{\partial x} + \mu \nabla^2 u_1 - \gamma \frac{\partial T}{\partial x} + \mu_0 \sigma_0 H_0 (E_2 - \mu_0 H_0 \frac{\partial u_1}{\partial t}) - \kappa_0 H_0 \mu_0 \frac{\partial T}{\partial y} = \rho \frac{\partial^2 u_1}{\partial t^2}, \quad (2.10)$$

$$(\lambda + \mu) \frac{\partial \Sigma}{\partial y} + \mu \nabla^2 u_2 - \gamma \frac{\partial T}{\partial y} - \mu_0 \sigma_0 H_0 (E_1 + \mu_0 H_0 \frac{\partial u_2}{\partial t}) + \kappa_0 H_0 \mu_0 \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u_2}{\partial t^2}. \quad (2.11)$$

The following non-dimensional variables are used

$$x' = c_1 \eta x, \quad y' = c_1 \eta y, \quad u'_1 = c_1 \eta u_1, \quad u'_2 = c_1 \eta u_2, \quad t' = c_1^2 \eta t, \quad \tau' = c_1^2 \eta \tau, \quad \psi = \frac{\gamma T}{\lambda + 2\mu},$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\mu}, \quad h' = \frac{\eta h}{\sigma_0 \mu_0 H_0}, \quad E'_i = \frac{\eta E_i}{\sigma_0 \mu_0^2 H_0 c_1}, \quad \kappa'_0 = \frac{\mu_0 H_0 \kappa}{\gamma}.$$

Using these non-dimensional terms, in the equations of motion and heat conduction equation are represented by

$$(\beta^2 - 1) \Sigma_{,x} + \nabla^2 u_1 - \beta^2 \psi_{,x} - \beta^2 \kappa_0 \psi_{,y} + \beta^2 \alpha \beta_1 (\beta_1 E_2 - u_{1,t}) = \beta^2 u_{1,tt}, \quad (2.12)$$

$$(\beta^2 - 1) \Sigma_{,y} + \nabla^2 u_2 - \beta^2 \psi_{,y} + \beta^2 \kappa_0 \psi_{,x} - \beta^2 \alpha \beta_1 (\beta_1 E_1 + u_{2,t}) = \beta^2 u_{2,tt}, \quad (2.13)$$

$$\nabla^2 \psi = (\psi_{,t} + \tau_0 \psi_{,tt}) + \epsilon_1 (\Sigma_{,t} + \tau_0 \Sigma_{,tt}). \quad (2.14)$$

2.2 Wave propagation

For a plane harmonic wave, the displacement components and temperature field which are the solution of Eqs. (2.12) – (2.14) take the following form (Ben-Menahem and Singh, 1981)

$$\{u_1, u_2, \psi\} = \{A, B, C\} e^{i\{ \omega t - k(x \sin \theta_0 + y \cos \theta_0) \}}, \quad (2.15)$$

where A , B , C are amplitudes, k is dimensionless complex wavenumber, ω is dimensionless angular frequency. If we take $k = \mathcal{R}(k) + i\mathcal{I}(k)$, then $\mathcal{R}(k)$ and $\mathcal{I}(k)$ denote the real and imaginary parts of k respectively. For the existence of waves in real:

$$\mathcal{R}(k) > 0, \quad \mathcal{I}(k) \leq 0.$$

It may be noted that the phase speed of the wave will be represented by $\frac{\omega}{\mathcal{R}(k)}$, while $\mathcal{I}(k)$ gives attenuation coefficient.

Using Eqs. (2.12)-(2.15), we get

$$(k^2 a_{11} + a_{12})A + k^2 b_{11}B + k c_{11}C = 0, \quad (2.16)$$

$$k^2 b_{11}A + (k^2 b_{21} + a_{12})B + k c_{21}C = 0, \quad (2.17)$$

$$k a_3 A + k b_3 B + (k^2 + \alpha_0)C = 0, \quad (2.18)$$

where

$$\begin{aligned} a_{11} &= -(\beta^2 \sin^2 \theta_0 + \cos^2 \theta_0), \quad a_{12} = \omega \beta^2 (\imath \alpha \beta_1^2 H_0 \mu_0 - \imath \alpha \beta_1 + \omega), \quad b_{11} = \sin \theta_0 \cos \theta_0 (1 - \beta^2), \\ c_{11} &= \imath \beta^2 (\sin \theta_0 + \kappa_0 \cos \theta_0), \quad b_{21} = -(\sin^2 \theta_0 + \beta^2 \cos^2 \theta_0), \quad c_{21} = \imath \beta^2 (\cos \theta_0 - \kappa_0 \sin \theta_0), \\ a_3 &= -\imath \epsilon_1 \sin \theta_0 \alpha_0, \quad b_3 = -\imath \epsilon_1 \cos \theta_0 \alpha_0, \quad \alpha_0 = \imath \omega - \tau \omega^2. \end{aligned}$$

The non-singular solutions of (2.16)-(2.18) give

$$k^6 l_0 + k^4 l_1 + k^2 l_2 + l_3 = 0, \quad (2.19)$$

where

$$\begin{aligned} l_0 &= b_{21} a_{11} - b_{11}^2, \quad l_2 = a_{12} a_{11} \alpha_0 + b_{21} \alpha_0 a_{12} + a_{12}^2 - b_3 c_{21} a_{12} - c_{11} a_3 a_{12}, \quad l_3 = a_{12}^2 \alpha_0, \\ l_1 &= b_{21} \alpha_0 a_{11} + a_{11} a_{12} - b_3 c_{21} a_{11} + b_{21} a_{12} - b_{11}^2 \alpha_0 + b_{11} a_3 c_{21} + c_{11} b_3 b_{11} - a_3 b_{21} c_{11}. \end{aligned}$$

If V_1 , V_2 and V_3 are the phase velocities corresponding to longitudinal, transverse and thermal waves respectively, then these values will satisfy Eq. (2.19) and $V_1 > V_2 > V_3$. We consider a generalized magneto-thermoelastic half-space, ($x \geq 0$) with a constant magnetic intensity, $H = (0, 0, H_0)$ which acts parallel to the bounding plane. The full form of displacement components and temperature field are given by

$$(u_1, u_2, \psi) = (1, f_0, r_0) A_0 e^{\imath \{ \omega t - k_0 (x \sin \theta_0 + y \cos \theta_0) \}} + \sum_{i=1}^3 (1, f_i, r_i) A_i e^{\imath \{ \omega t - k_i (x \sin \theta_i + y \cos \theta_i) \}}, \quad (2.20)$$

where A_i is amplitude constant, f_i and r_i are the coupling parameters whose expressions are given below

$$\begin{aligned} f_0 &= \frac{(k_i^2 + \alpha_0)(k_i a_3 b_{21} + a_{12} a_3 - k_i^2 b_3 b_{11} - k_i^2 b_{21} a_3 b_3 - a_3 b_3 a_{12}) + a_3 b_3 k_i^2 c_{21}}{b_3 \{ (k_i^2 + \alpha_0)(k_i^2 b_{21} + a_{12}) - k_i^2 b_3 c_{21} \}}, \\ f_i &= -\frac{(k_i^2 + \alpha_0)(k_i a_3 b_{21} + a_{12} a_3 - k_i^2 b_3 b_{11} - k_i^2 b_{21} a_3 b_3 - a_3 b_3 a_{12}) + a_3 b_3 k_i^2 c_{21}}{b_3 \{ (k_i^2 + \alpha_0)(k_i^2 b_{21} + a_{12}) - k_i^2 b_3 c_{21} \}}, \end{aligned}$$

$$r_0 = -\frac{k_i^3 b_3 b_{11} + k_i a_3 (k_i^2 b_{21} + a_{12})}{(k_i^2 + \alpha_0)(k_i^2 b_{21} + a_{12}) + k_i^2 b_3 c_{21}}, \quad r_i = \frac{k_i^3 b_3 b_{11} - k_i a_3 (k_i^2 b_{21} + a_{12})}{(k_i^2 + \alpha_0)(k_i^2 b_{21} + a_{12}) - k_i^2 b_3 c_{21}}.$$

The Snell's law, for the problem, is given by

$$k_0 \sin \theta_0 = k_i \sin \theta_i, \quad (i = 1, 2, 3). \quad (2.21)$$

2.3 Appropriate Boundary Conditions

The normal and tangential stress tensors and gradient of temperature field at $y = 0$ vanish. These conditions may be written as: At $y = 0$

- (i) $\sigma_{22} + \tau_{22} = 0$,
- (ii) $\sigma_{21} + \tau_{21} = 0$ and
- (iii) Temperature gradient = 0.

These conditions can be written as

$$(\mu_0 H_0^2 + \beta^2 - 2)u_{1,x} + (\mu_0 H_0^2 + \beta^2)u_{2,y} - \beta^2 \psi = 0, \quad (2.22)$$

$$\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0. \quad (2.23)$$

Using Eqs. (2.20) and (2.21) into (2.22) and (2.23), we get a matrix equation

$$[B_{ij}][Z] = [N], \quad (i, j = 1, 2, 3) \quad (2.24)$$

where B_{ij} is 3×3 matrix, $Z = [Z_1, Z_2, Z_3]^t$ is the amplitude ratio and $N = [N_1, N_2, N_3]^t$. The expression of $Z_i = \frac{A_i}{A_0}$ are given by

$$\begin{aligned} Z_1 &= \frac{N_1(B_{11}B_{33} - B_{23}B_{32}) - B_{21}(N_2B_{33} - N_3B_{22}) + B_{31}(N_2B_{23} - N_3B_{22})}{B_{11}(B_{22}B_{33} - B_{32}B_{23}) - B_{12}(B_{21}B_{33} - B_{31}B_{23}) + B_{13}(B_{21}B_{32} - B_{31}B_{22})}, \\ Z_2 &= \frac{B_{11}(N_2B_{33} - N_3B_{32}) - N_1(B_{12}B_{33} - B_{13}B_{32}) + B_{31}(B_{12}N_3 - B_{13}N_2)}{B_{11}(B_{22}B_{33} - B_{32}B_{23}) - B_{12}(B_{21}B_{33} - B_{31}B_{23}) + B_{13}(B_{21}B_{32} - B_{31}B_{22})}, \\ Z_3 &= \frac{B_{11}(B_{22}N_3 - B_{23}N_2) - B_{21}(B_{12}N_3 - B_{13}N_2) + N_1(B_{12}B_{23} - B_{13}B_{22})}{B_{11}(B_{22}B_{33} - B_{32}B_{23}) - B_{12}(B_{21}B_{33} - B_{31}B_{23}) + B_{13}(B_{21}B_{32} - B_{31}B_{22})}, \end{aligned} \quad (2.25)$$

where the expressions of B_{ij} and N_i are given below

$$\begin{aligned} B_{1j} &= i\{(\mu_0 H_0^2 + \beta^2 - 2)k_0 \sin \theta_0 - (\mu_0 H_0^2 + \beta^2)f_j k_j \cos \theta_j\} + \beta^2 r_i, \\ B_{3j} &= r_j k_j \cos \theta_j, \quad N_1 = -i\{(\mu_0 H_0^2 + \beta^2 - 2)k_0 \sin \theta_0 + f_0(\mu_0 H_0^2 + \beta^2)k_0 \cos \theta_0\} - \beta^2 r_0, \\ N_2 &= k_0(\cos \theta_0 + f_0 \sin \theta_0), \quad N_3 = r_0 k_0 \cos \theta_0, \quad B_{2j} = k_j \cos \theta_j - f_j k_0 \sin \theta_0. \end{aligned}$$

It may be noted that these ratios depend on elastic constants, angle of propagation, magnetic and thermal parameters.

2.4 Energy Partition

The incident energy is distributed among the reflected waves at the boundary surface. The power per unit area represents the energy flux across the surface element and is given by

$$\mathcal{P}_{ij} = \frac{1}{2} \mathcal{R}(< (\sigma_{22} + \tau_{22})^i, (\dot{u}_2^j)^* >) + \frac{1}{2} \mathcal{R}(< (\sigma_{21} + \tau_{21})^i, (\dot{u}_1^j)^* >) \quad (2.26)$$

where $()^*$ denotes complex conjugate of $()$.

There are bulk energy and interacting energy between the incident and reflected waves. This energy can be obtained together from the energy matrix, $\mathcal{E}_{ij}$ in the form of ratios as

$$\mathcal{E}_{ij} = -\frac{\mathcal{P}_{ij}}{\mathcal{P}_{00}}, \quad (i, j = 1, 2, 3) \quad (2.27)$$

where

$$\mathcal{P}_{00} = \frac{|A_0|^2 \omega k_0}{2} (\cos \theta_0 \{|f_0|^2 (\mu_0 H_0^2 + \beta^2) + 1\} - \sin \theta_0 \{f_0^* (\mu_0 H_0^2 + \beta^2 - 2) + f_0\}),$$

$$\mathcal{P}_{ij} = \frac{A_i A_j^* \omega k_i}{2} (\cos \theta_i \{(\mu_0 H_0^2 + \beta^2) f_j^* f_i + 1\} - \sin \theta_i \{f_j^* (\mu_0 H_0^2 + \beta^2 - 2) - f_i\}).$$

The sum of the net interacting energy, $\mathcal{E}^{int}$ and bulk energy ratios, $\mathcal{E}^{bulk}$ due to the interactions of incident wave with the three dissimilar reflected waves can be obtained respectively from

$$\mathcal{E}^{int} = \sum_{i=1}^3 (\mathcal{E}_{0i} + \mathcal{E}_{i0}) + \sum_{i=1}^3 (\sum_{j=1}^3 \mathcal{E}_{ij} - \mathcal{E}_{ii}), \quad \mathcal{E}^{bulk} = \sum_{i=1}^3 \mathcal{E}_{ii}. \quad (2.28)$$

Thus, the energy ratios corresponding to reflected longitudinal, transverse and thermal waves are respectively represented by E_1 , E_2 and E_3 which are given by

$$E_i = \mathcal{E}_i^{int} + \mathcal{E}_i^{bulk}. \quad (2.29)$$

The sum of energy ratios at the boundary surface for the three reflected waves must be equal to unity.

$$\mathcal{E}^{bulk} + \mathcal{E}^{int} = \sum_{i=1}^3 E_i = 1. \quad (2.30)$$

This shows the conservation law of energy for this specific problem.

2.5 Special cases

Case 1: If we neglect magnetic effect, the problem reduces to wave propagation in generalized thermoelastic material. In this case, $h = H_0 = \alpha = 0$ and $a_{12} = \omega^2 \beta^2$. The amplitude and energy ratios are given by Eqs.(2.25) and (2.27) respectively with the following modified values

$$\begin{aligned} N_1 &= -i\{\sin \theta_0 k_0(\beta^2 - 2) + f_0 k_0 \cos \theta_0 \beta^2\} - \beta^2 r_0, \\ B_{1j} &= i\{(\beta^2 - 2)k_0 \sin \theta_0 - \beta^2 f_j \cos \theta_j k_j\} + \beta^2 r_j, \quad (j = 1, 2, 3) \\ \mathcal{P}_{00} &= \frac{|A_0|^2 \omega k_0}{2} (\cos \theta_0 (|f_0|^2 \beta^2 + 1) - \sin \theta_0 \{f_0^* (\beta^2 - 2) + f_0\}), \\ \mathcal{P}_{ij} &= \frac{A_i A_j^* \omega k_i}{2} (\cos \theta_i (\beta^2 f_j^* f_i + 1) - \sin \theta_i \{f_j^* (\beta^2 - 2) - f_i\}). \end{aligned}$$

Case 2: If we neglect thermal effect, the problem reduces to wave propagation in isotropic material. In this case, $\kappa = \eta = C_e = \epsilon_1 = \beta_1 = \gamma = \alpha_T = 0$ and f_i is taken so that:

$$(\sin \theta_0 u_1^{(0)}, \cos \theta_0 u_2^{(0)}), \quad (\sin \theta_1 u_1^{(1)}, -\cos \theta_1 u_2^{(1)}), \quad (\cos \theta_2 u_1^{(2)}, \sin \theta_2 u_2^{(2)})$$

are components of the displacement. The amplitude and energy ratios are respectively given by

$$Z_1 = \frac{\sin 2\theta_0 \sin 2\theta_2 - \beta^2 \cos^2 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \beta^2 \cos^2 2\theta_2}, \quad Z_2 = \frac{2\beta \sin 2\theta_0 \cos 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \beta^2 \cos^2 2\theta_2}, \quad (2.31)$$

$$E_1 = Z_1^2, \quad E_2 = \frac{\cos \theta_2 Z_2^2}{\beta^2 \cos \theta_0}. \quad (2.32)$$

These results exactly match with those of Achenbach(1976) for classical elasticity.

2.6 Numerical Computations

We have computed the amplitude and energy ratios of reflected waves for the longitudinal wave, transverse wave and thermal wave through Matlab programming. The following relevant values of parameters for the generalized thermoelastic solid are considered (Ezzat and Youssef, 2005)

Symbols	Values	Symbols	Values
κ_0	386 N/Ks	α_T	$1.78 \times 10^{-5} K^{-1}$
C_e	$383.1 m^2/Ks^2$	η	$8886.73 m/s^2$
μ	$3.86 \times 10^{10} N/m^2$	λ	$7.76 \times 10^{10} N/m^2$
c_1	1.39×10^{-5}	ρ	$8954 kg/m^3$
μ_0	$4\pi \times 10^{-7} Nm s^2/C^2$	ϵ_0	$10^{-9}/36\pi C^2/Nm^2$
H_0	$10^3 C/ms$		

Table 2.2: Values of the parameters

It is interesting to note the effects of initial magnetic field H_0 and specific heat C_e on the phase velocity, amplitude and energy ratios of the reflected waves. These effects lead to understand the physical properties of the materials. The variation of phase velocities of longitudinal, transverse and thermal waves with angular frequency are computed for different values of H_0 and C_e in Figures 2.1, 2.2 and 2.3 respectively. In Figure 2.1, the phase velocity of longitudinal wave increases with the increase of ω thereby making a parabolic profile. It is observed that the values of V_1 decrease with the increase of H_0 and C_e . The phase velocity of the transverse wave in Figure 2.2 with the angular frequency shows a parabolic-type decreasing profile until it reaches a minimum value located in the range of $30Hz - 50Hz$ to then grow again.

The minimum effect of H_0 is observed in the lower values of ω . Figure 2.3 shows that V_3 decreases with the increase of ω . It is observed that the values of V_3 decrease with the increase of C_e and H_0 .

The variation of amplitude and energy ratios with angle of incidence are plotted for different values of H_0 and C_e in Figures (2.4) – (2.10). In Figure 2.4, the amplitude ratio, Z_1 corresponding to longitudinal reflected wave starts from certain value and decreases upto certain value of θ_0 which increases thereafter with the increase of θ_0 .

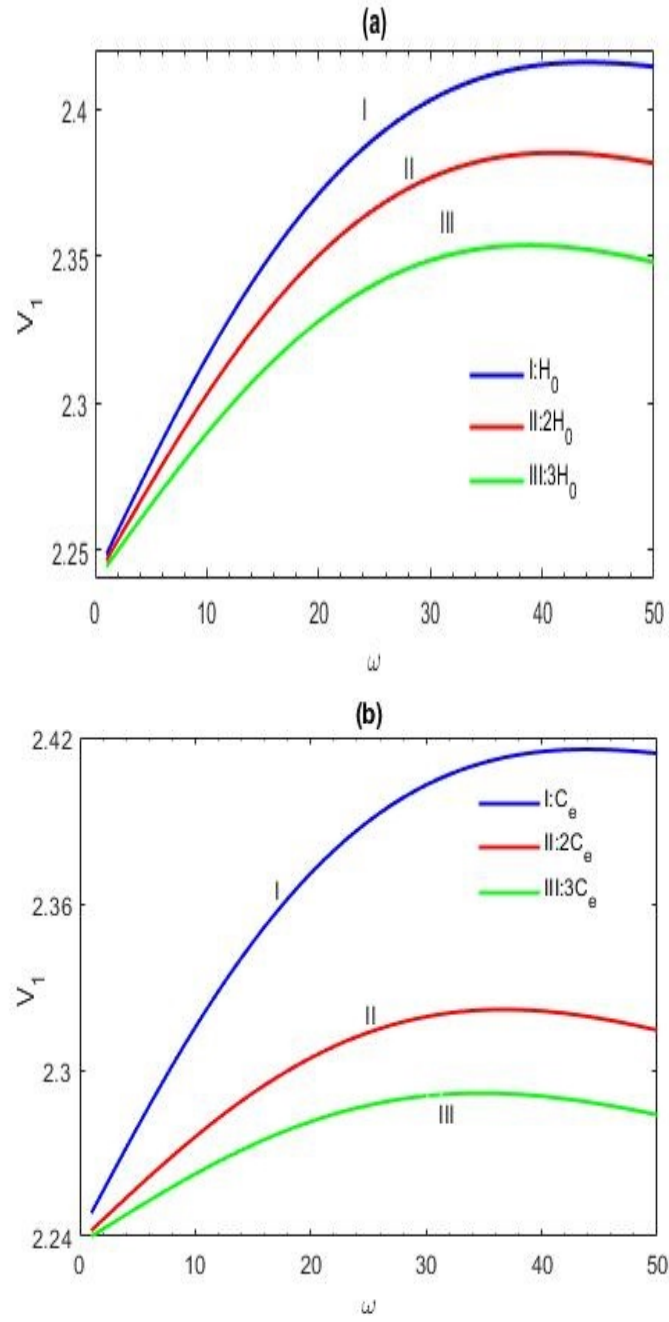


Figure 2.1: Variation of V_1 for different values of (a) H_0 and (b) C_e

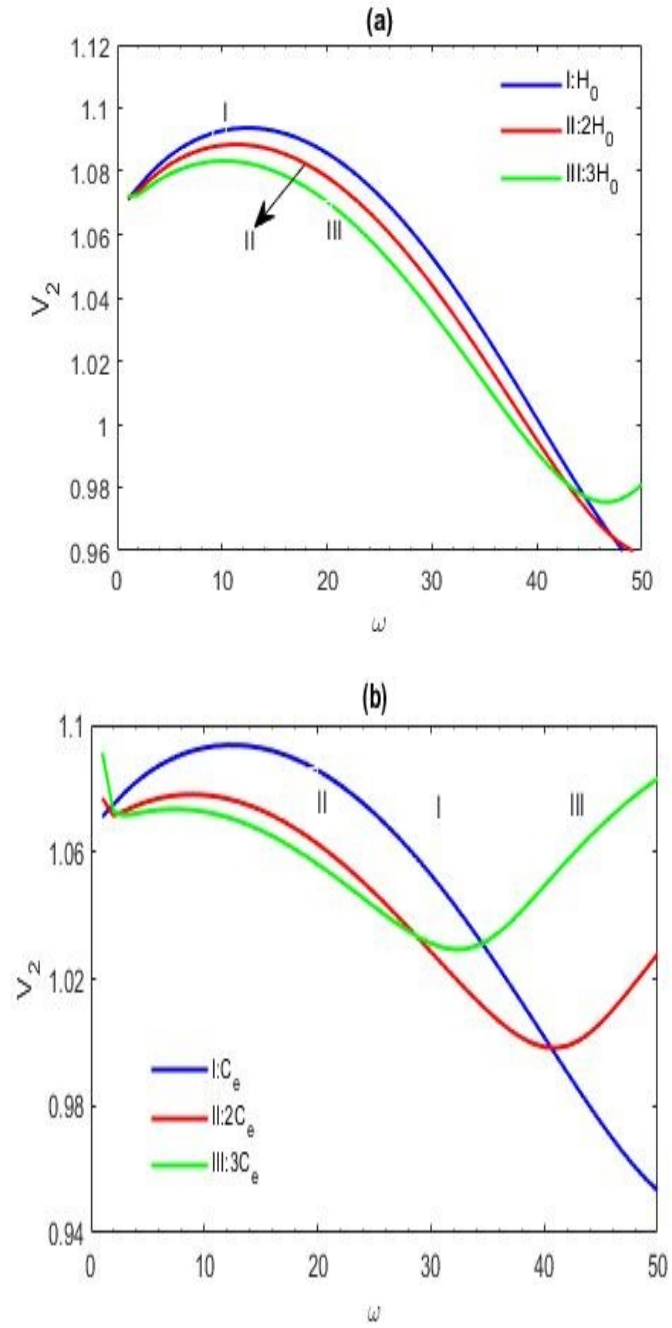
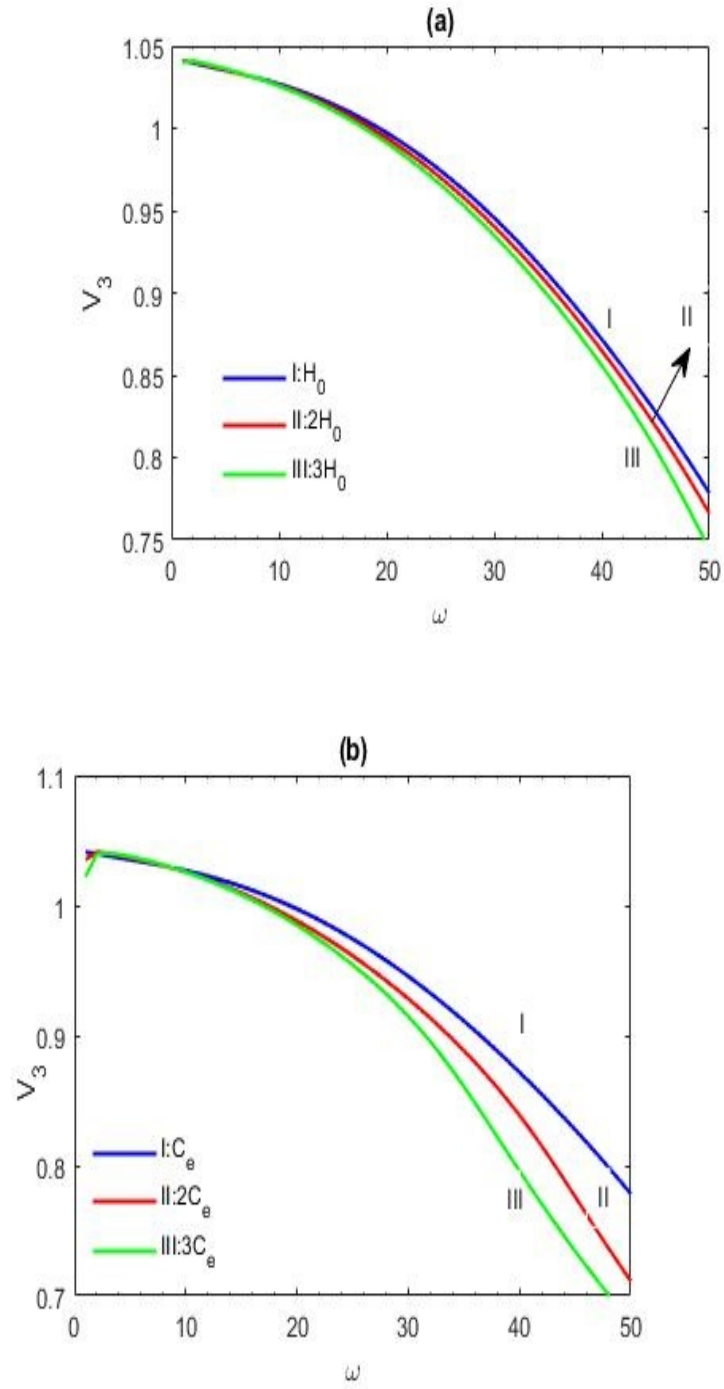
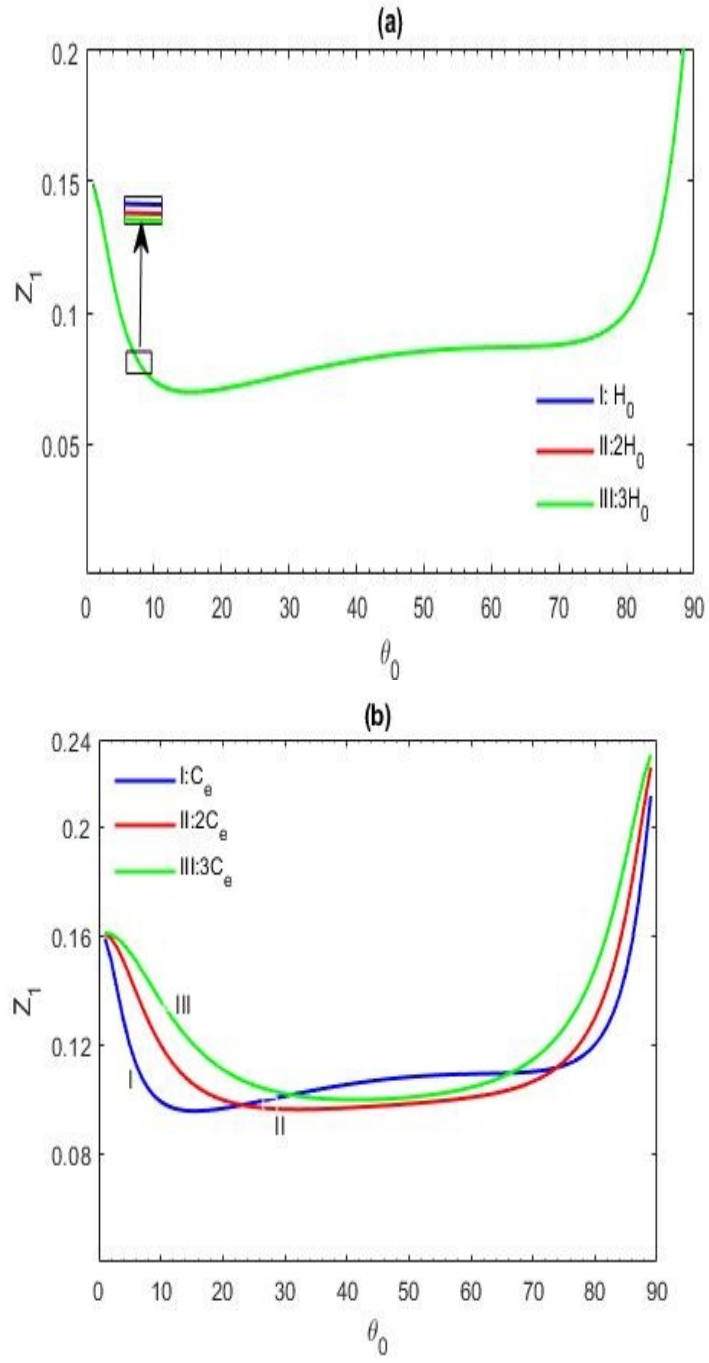
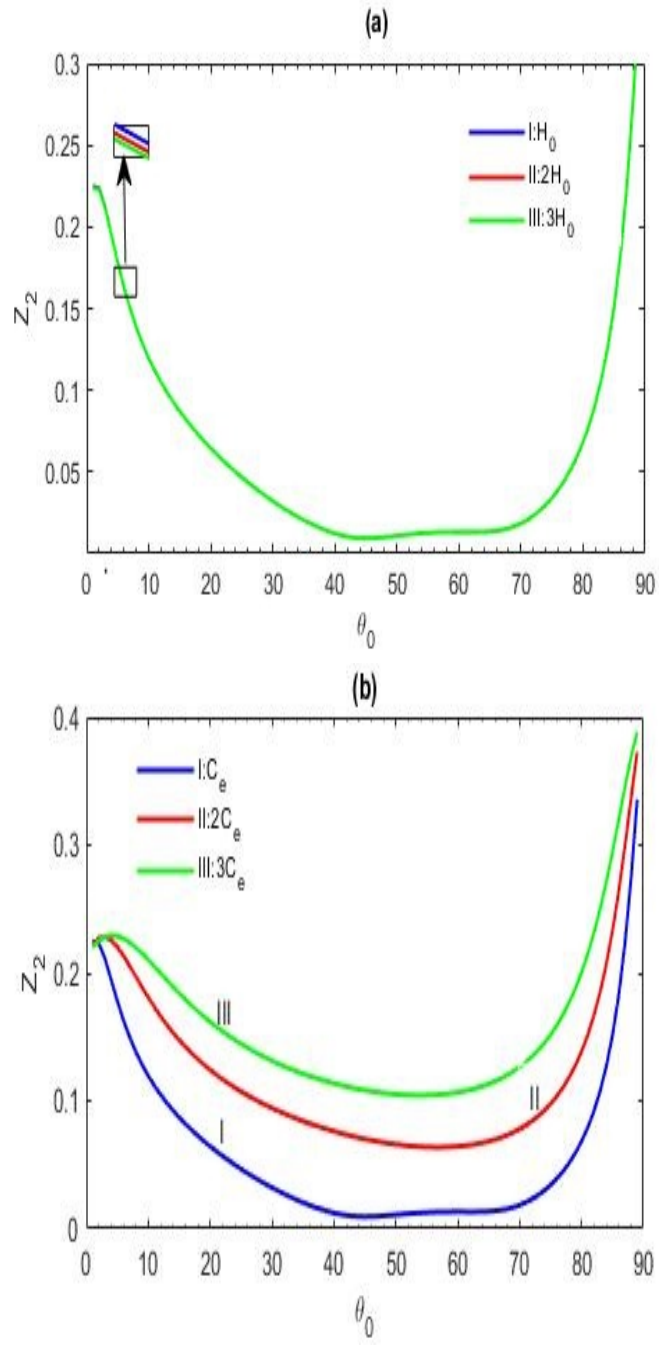
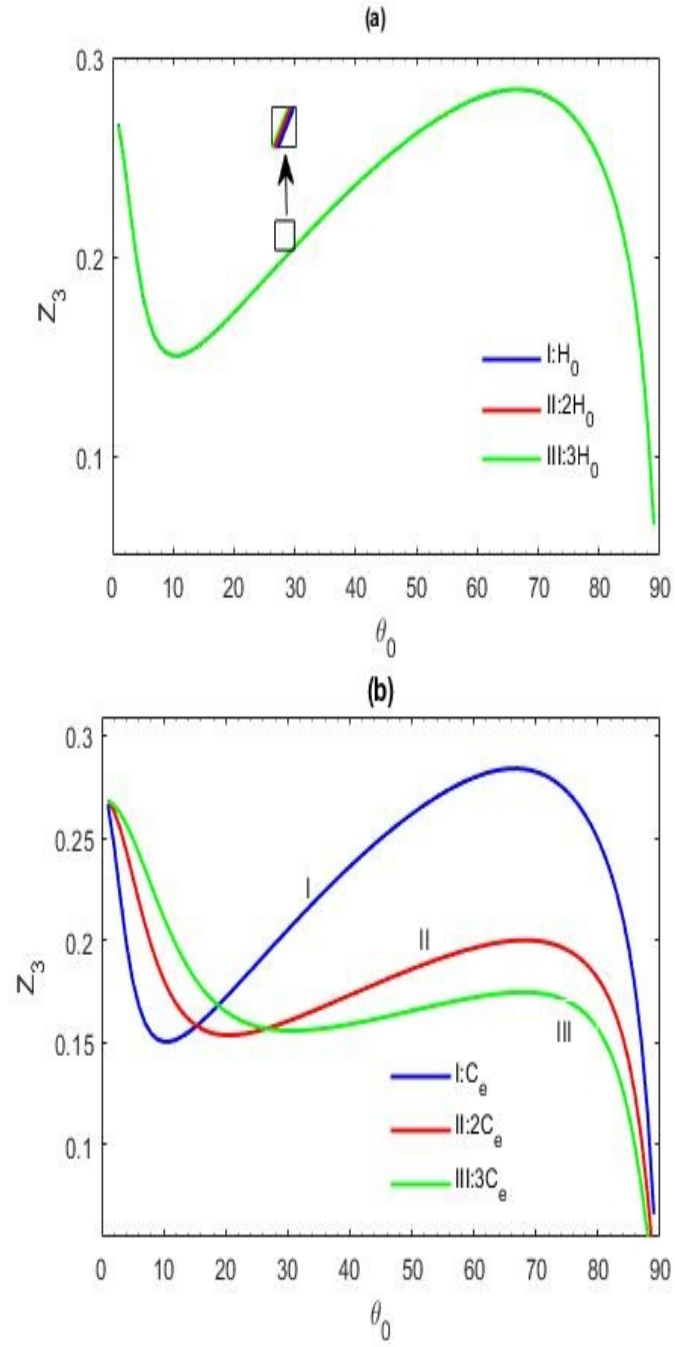


Figure 2.2: Variation of V_2 for different values of (a) H_0 and (b) C_e

Figure 2.3: Variation of V_3 for different values of (a) H_0 and (b) C_e

Figure 2.4: Variation of Z_1 for different values of (a) H_0 and (b) C_e

Figure 2.5: Variation of Z_2 for different values of (a) H_0 and (b) C_e

Figure 2.6: Variation of Z_3 for different values of (a) H_0 and (b) C_e

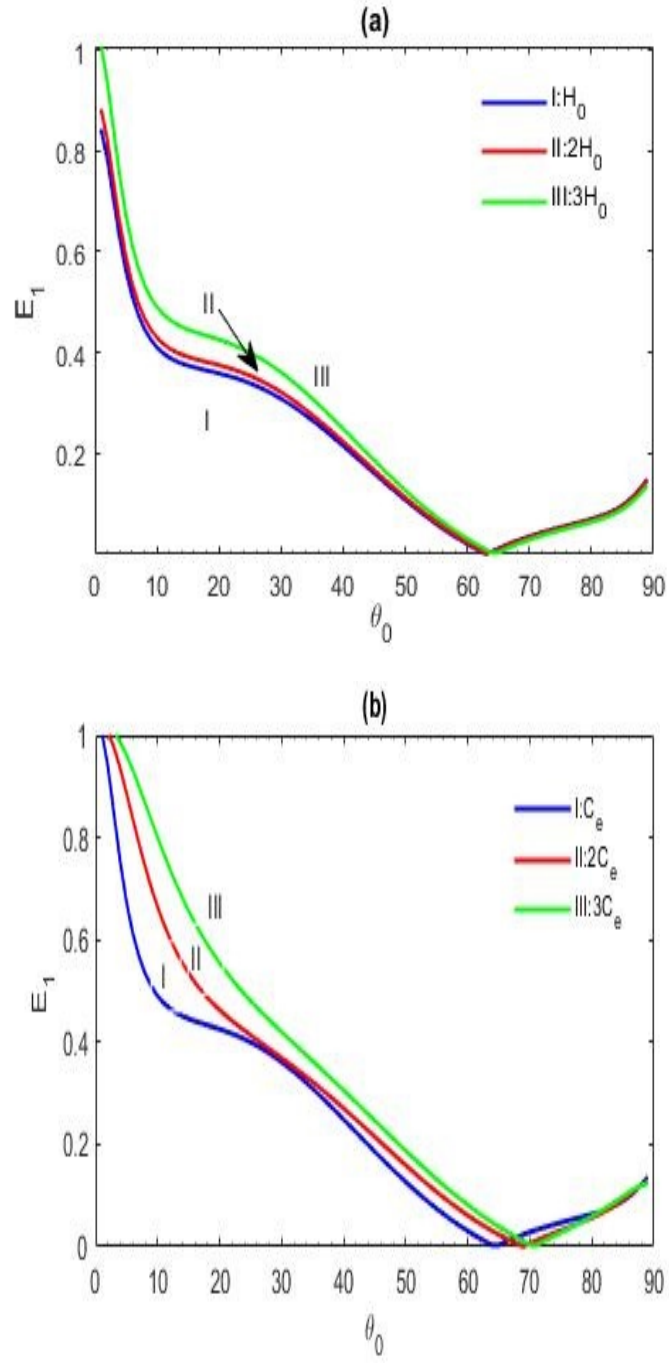


Figure 2.7: Variation of E_1 for different values of (a) H_0 and (b) C_e

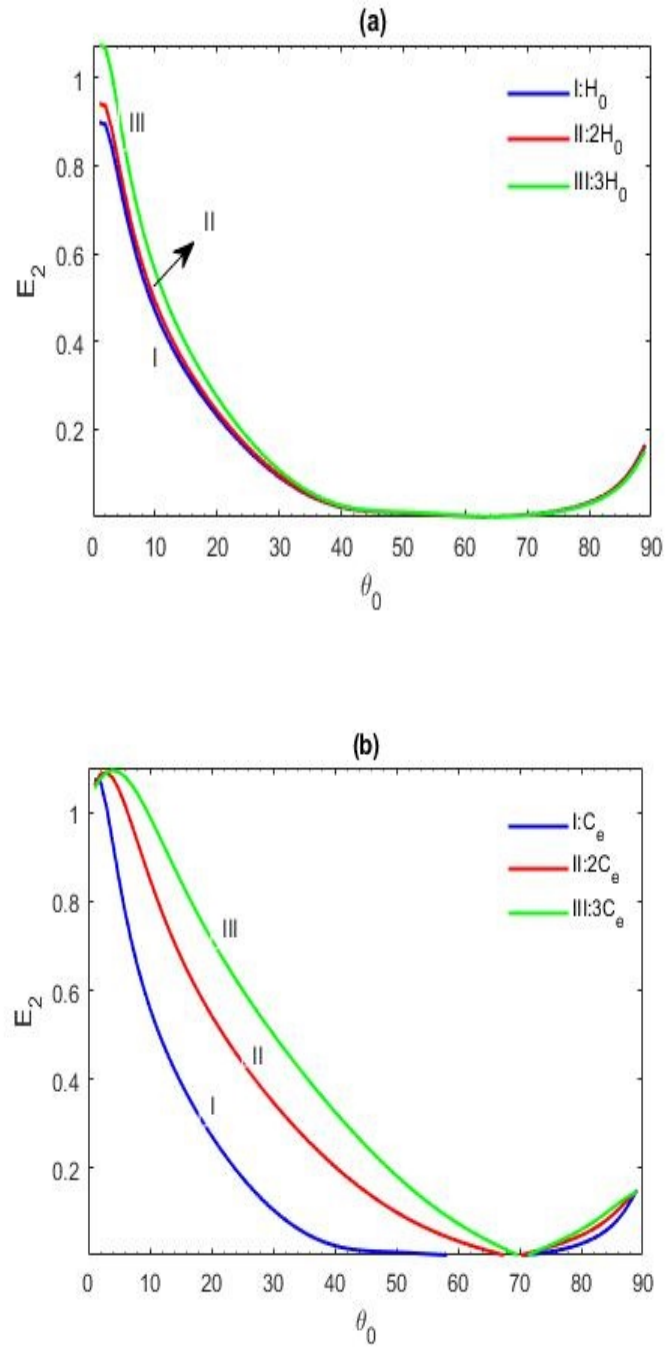


Figure 2.8: Variation of E_2 for different values of (a) H_0 and (b) C_e

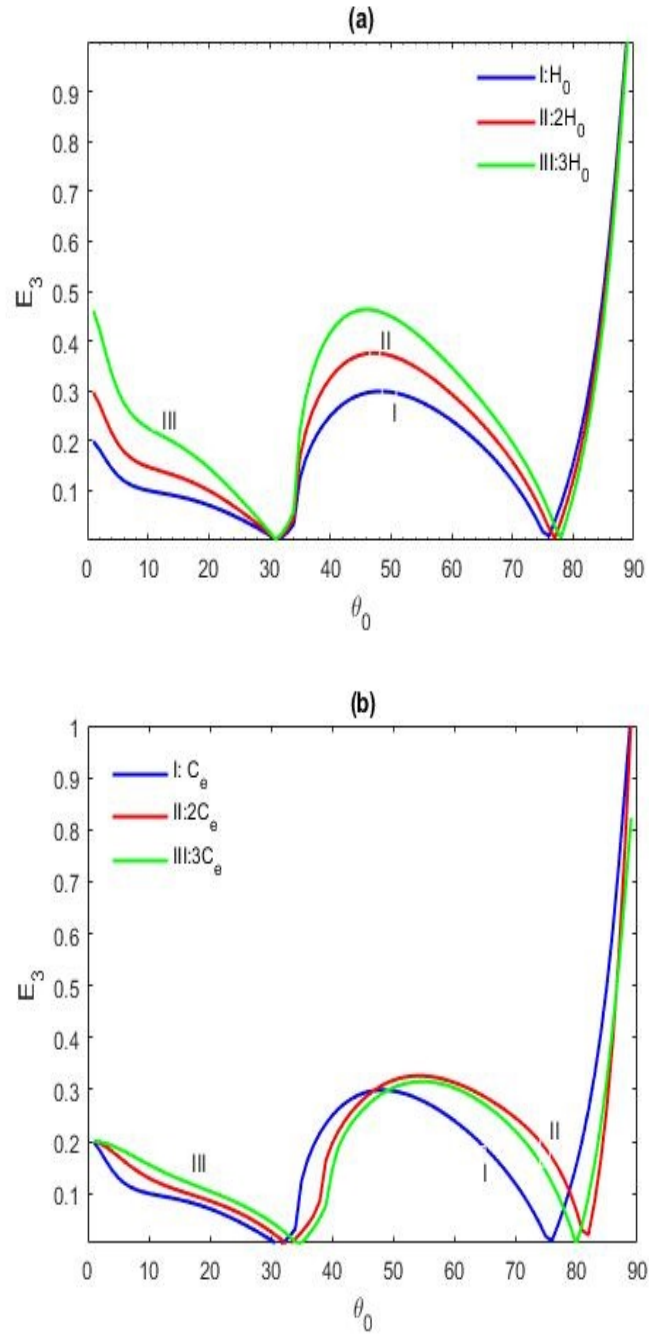


Figure 2.9: Variation of E_3 for different values of (a) H_0 and (b) C_e

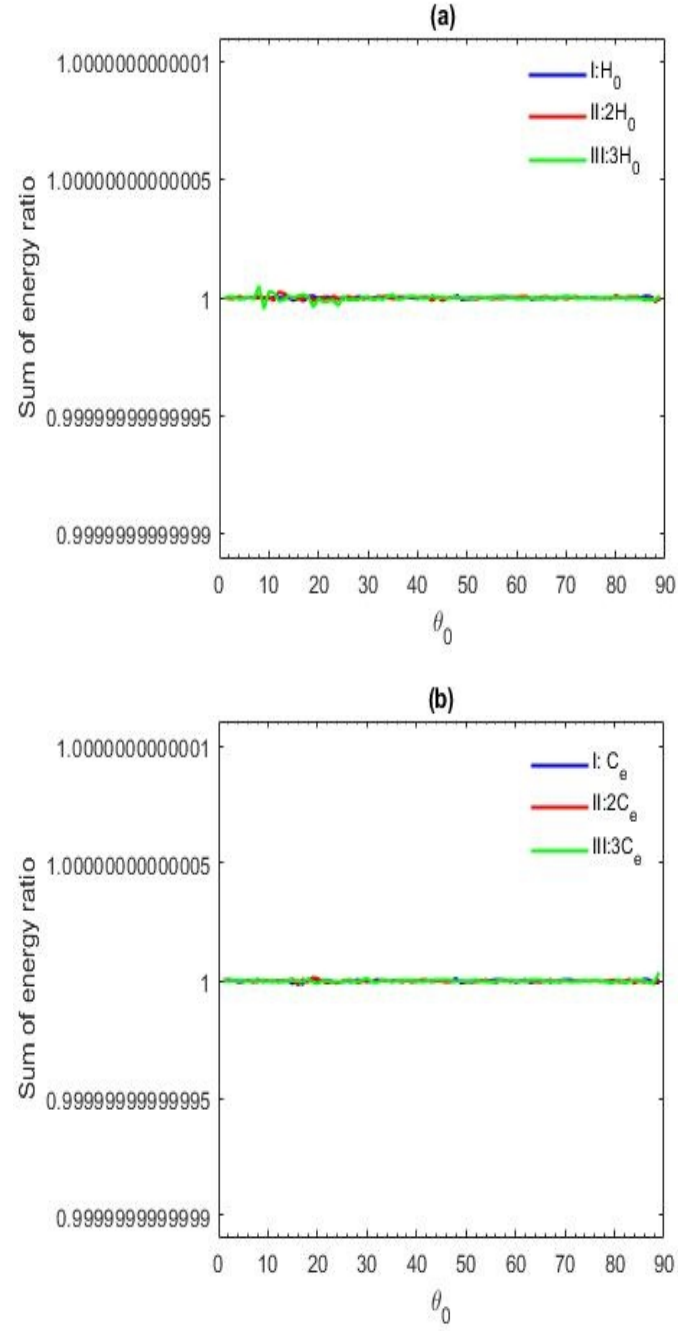


Figure 2.10: Sum of energy ratios for different values of (a) H_0 and (b) C_e

The variation of Z_2 is similar with Z_1 . In Figure 2.6, Z_3 starts from certain value which decreases followed by increasing and then decreasing with the increase of θ_0 . We have seen small effect of H_0 on Z_i and the effect of C_e on Z_i are minimum near the normal angle of incidence. In Figures 2.7 and 2.8, the energy ratios E_1 and E_2 start from certain values which decrease to zero at certain values of $\theta_0 = \theta_g$ followed by increasing thereafter with the increase of θ_0 . The value of θ_g for E_1 and E_2 are different for different values of H_0 and C_e . The values of θ_g for different values of H_0 and C_e on E_1 are near to 64° and 70° respectively. This value is near 70° for the energy ratio, E_2 .

In Figure 2.9, E_3 starts from certain value which decreases to zero making a parabolic curves in the regions $I : 31^\circ \leq \theta_0 \leq 74^\circ$, $II : 31^\circ \leq \theta_0 \leq 76^\circ$, $III : 32^\circ \leq \theta_0 \leq 78^\circ$ in Figure 2.9(a) and $I : 32^\circ \leq \theta_0 \leq 75^\circ$, $II : 34^\circ \leq \theta_0 \leq 82^\circ$, $III : 35^\circ \leq \theta_0 \leq 80^\circ$ in Figure 2.9(b). Then, the graph of E_3 increases with the increase of θ_0 . Interestingly, these parabolic regions are different for different values of H_0 and C_e . The effect of H_0 and C_e on E_i are found to be minimum near grazing angle of incidence. In Figure 2.10, we have seen that sum of energy ratios of the reflected waves is closed to unity. This shows the conservation law of energy for this problem.

2.7 Conclusions

We have investigated the problem of reflection of waves from a half-space of generalized magneto thermoelastic materials. The expressions of the amplitude and energy ratios of reflected waves are found by using the appropriate boundary conditions. These ratios and phase velocities of the longitudinal, transverse and thermal waves are calculated numerically and the results are plotted graphically. We have observed that the present study confirms the significant effects of the magnetic parameter and thermal parameter on the wave propagation. The following points are the concluding remarks:

- The amplitude and energy ratios are functions of angle of incidence, magnetic, thermal and elastic parameters.
- The phase velocity of longitudinal wave increases with the increase of H_0 and C_e .
- The phase velocity corresponding to thermal wave increases with the increase of ω .
- The variations of Z_1 and Z_2 are similar in nature.

- The effect of H_0 on Z_i is very small.
- The effects of H_0 and C_e on E_i are minimum near the grazing angle of incidence.
- The sum of energy ratios of reflected waves are close to one.

Chapter 3

3D Analysis of Wave Propagation in Generalized Magneto Thermoelastic Medium²

3.1 Introduction

The analysis of three dimensional thermoelastic problem has been found relatively less attention due to complicated form of the solution. Youssef(2005) investigated the problem of generalized thermoelasticity with one relaxation time in an isotropic elastic medium with temperature-dependent mechanical and thermal properties. He considered the modulus of elasticity and the thermal conductivity as a linear function of temperature. Tzou(1995) proposed a universal constitutive equation between the heat flux vector and the temperature gradient to cover the fundamental behaviours of diffusion (macroscopic in both space and time), wave (macroscopic in space but microscopic in time), phonon electron interactions(microscopic in both space and time), and pure phonon scattering. This model is generalized from the dual-phase-lag concept accounting for the lagging behaviour in the high-rate response.

In this chapter, we investigate the three dimensional problem of wave propagation in generalized magneto thermoelastic medium. The eigenvalue approach is adopted to solve the vector matrix differential equation which represents the equation of motion for the generalized thermoelastic materials. Stress tensors, temperature field and components of displacement are obtained theoretically and numerically with the help of appropriate boundary conditions. The effect of magnetic and relaxation time on the propagation of waves are critically examined.

²Boruah S. and Singh, S.S.(2024). "3D Analysis of Wave Propagation in Generalized Magneto Thermoelastic Medium", *Sci Tech J*, **12(3)**, 38-44.

3.2 Basic Equations

Using $\vec{\mathbf{H}} = (H_x, H_y, 0)$, $\vec{\mathbf{E}} = (E_x, E_y, E_z)$ and $\vec{\mathbf{u}} = (u, v, w)$ into Eqs. (1.21) and (1.25), we get

$$-\mu_0 \frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \quad \mu_0 \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}, \quad (3.1)$$

where

$$\begin{aligned} E_x &= \frac{-1}{\sigma} \frac{\partial H_y}{\partial z} + \mu_0 H_y \frac{\partial w}{\partial t}, \quad E_y = \frac{1}{\sigma} \frac{\partial H_x}{\partial z} - H_x \mu_0 \frac{\partial w}{\partial t}, \\ E_z &= \frac{1}{\sigma} \frac{\partial H_y}{\partial x} - \frac{1}{\sigma} \frac{\partial H_x}{\partial y} + \mu_0 H_x \frac{\partial v}{\partial t} - \mu_0 H_y \frac{\partial u}{\partial t} - \mu_0 \frac{\partial H_y}{\partial t}. \end{aligned}$$

The components of $\vec{\mathbf{J}}$ are given by using Eq.(1.25) as

$$J_x = \sigma(E_x - \mu_0 H_y \frac{\partial w}{\partial t}), \quad J_y = \sigma(E_y + \mu_0 H_x \frac{\partial w}{\partial t}), \quad J_z = \sigma(\mu_0 H_y \frac{\partial u}{\partial t} - \mu_0 H_x \frac{\partial v}{\partial t} + E_z).$$

Equation (3.1) is linearized by inserting $H_x = H_0 + h_x$ and $H_y = H_0 + h_y$ so that h_x and h_y denote change in the basic magnetic field, H_0 along the x and y directions respectively. These changes can be obtained using Eq.(3.1) as

$$h_x = -H_0 \left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} + \frac{\partial w}{\partial z} \right), \quad h_y = -H_0 \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} + \frac{\partial w}{\partial z} \right). \quad (3.2)$$

The constitutive relations of generalised thermoelastic solid are given by (Ezzat and Ellal, 2009) as

$$\bar{\sigma}_{ij} = H_i h_j + H_j h_i - (\mathbf{H} \cdot \mathbf{h}) \delta_{ij}, \quad (3.3)$$

The equations of motion and heat conduction equations for homogeneous magneto thermoelastic materials are derived using Eqs.(1.28) and (2.1) as

$$\begin{aligned} (c_3^2 - c_2^2)u_{xx} + (c_1^2 + c_2^2)u_{yy} + c_1^2 u_{zz} + c_2^2(v_{xx} - v_{yy}) + c_3^2 v_{yx} + (c_3^2 - c_2^2)w_{xz} - c_2^2 w_{zy} - c_4^2 T_x \\ = u_{tt}, \end{aligned} \quad (3.4)$$

$$c_2^2(u_{xx} - u_{yy}) + (c_1^2 - c_2^2)v_{xx} + (c_2^2 + c_3^2)v_{yy} + c_3^2 u_{xy} + c_1^2 v_{zz} + (c_2^2 + c_3^2)w_{yz} - c_4^2 T_y = v_{tt}, \quad (3.5)$$

$$c_1^2(w_{xx} + w_{yy}) + (c_3^2 + 2c_2^2)w_{zz} + (c_2^2 + c_3^2)u_{xz} - c_2^2 u_{yz} + (c_2^2 + c_3^2)v_{yz} - c_4^2 T_z = w_{tt}, \quad (3.6)$$

$$\kappa_0 T_{ii} = (T_{,t} + \tau_0 T_{,tt}) + \gamma T_0 (\Sigma_{,t} + \tau_0 \Sigma_{,tt}), \quad (3.7)$$

where $c_1^2 = \frac{\mu}{\rho}$, $c_2^2 = \frac{\mu_0 H_0^2}{\rho}$, $c_3^2 = \frac{\mu + \lambda}{\rho}$, $c_4^2 = \frac{\gamma}{\rho}$.

3.3 Solution of the problem

We assume that the solutions of Eqs.(3.4)-(3.7) take the following form

$$\{u, v, w, T\}(x, y, z, t) = \{\bar{u}, \bar{v}, \bar{w}, \bar{T}\}(z)e^{\iota(\alpha x + \beta y - \omega t)}, \quad (3.8)$$

where α, β are wave numbers along x and y directions respectively and ω is angular frequency.

Using Eq.(3.8) into the equations of motions and heat conduction equations (3.4)-(3.7), we have the following set of equations

$$\begin{aligned} \frac{d^2 \bar{u}}{dz^2} &= a_{51} \bar{u} + a_{52} \bar{v} + a_{54} \bar{T} + a_{57} \frac{d\bar{w}}{dz}, \\ \frac{d^2 \bar{v}}{dz^2} &= a_{61} \bar{u} + a_{62} \bar{v} + a_{64} \bar{T} + a_{67} \frac{d\bar{w}}{dz}, \end{aligned} \quad (3.9)$$

$$\begin{aligned} \frac{d^2 \bar{w}}{dz^2} &= a_{73} \bar{w} + a_{75} \frac{d\bar{u}}{dz} + a_{76} \frac{d\bar{v}}{dz} + a_{78} \frac{d\bar{T}}{dz}, \\ \frac{d^2 \bar{T}}{dz^2} &= a_{81} \bar{u} + a_{82} \bar{v} + a_{84} \bar{T} + a_{87} \frac{d\bar{w}}{dz}, \end{aligned} \quad (3.10)$$

$$\begin{aligned} a_{51} &= (c_{31}^2 - c_{21}^2)\alpha^2 + (1 + c_{21}^2)\beta^2 - k_1^2, \quad a_{52} = c_{21}^2(\alpha^2 - \beta^2) + c_{31}^2\alpha\beta, \\ a_{57} &= \iota\{\beta c_{21}^2 - (c_{31}^2 - c_{21}^2)\alpha\}, \quad a_{61} = \alpha\beta c_{31}^2 - (\beta^2 - \alpha^2)c_{21}^2, \quad a_{64} = \iota\beta c_{41}^2, \\ a_{87} &= \frac{T_0\omega(1 - \tau_0\omega\iota)c_{41}^2}{c_{51}^2}, \quad a_{67} = -\iota\beta(c_{21}^2 + c_{31}^2), \quad a_{73} = \frac{\alpha^2 + \beta^2 - k_1^2}{c_{31}^2 + 2c_{21}^2}, \\ a_{75} &= \iota \frac{c_{21}^2(\beta - \alpha) - \alpha c_{31}^2}{c_{31}^2 + 2c_{21}^2}, \quad a_{76} = \frac{-\iota\beta(c_{21}^2 + c_{31}^2)}{c_{31}^2 + 2c_{21}^2}, \quad a_{78} = \frac{c_{41}^2}{c_{31}^2 + 2c_{21}^2}, \\ a_{81} &= \frac{T_0\omega\alpha c_{41}^2(1 - \iota\tau_0\omega)}{c_{51}^2}, \quad a_{82} = \frac{T_0\omega\beta c_{41}^2(1 - \iota\tau_0\omega)}{c_{51}^2}, \quad a_{54} = \iota\alpha c_{41}^2, \\ a_{62} &= -k_1^2 + \beta^2(c_{31}^2 + c_{21}^2) - \alpha^2(c_{21}^2 - 1), \quad a_{84} = (\alpha^2 + \beta^2) - \iota \frac{c_{e1}(1 - \iota\tau_0\omega)\omega}{c_{51}^2}. \end{aligned}$$

These Equations (3.9) and (3.10) can now be written in the form of vector notation as

$$\frac{d\vec{V}}{dz} = A\vec{V}, \quad (3.11)$$

where

$$\vec{V} = [\bar{u}, \bar{v}, \bar{w}, \bar{T}, \frac{d\bar{u}}{dz}, \frac{d\bar{v}}{dz}, \frac{d\bar{w}}{dz}, \frac{d\bar{T}}{dz}]^T,$$

and

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ a_{51} & a_{52} & 0 & a_{54} & 0 & 0 & a_{57} & 0 \\ a_{61} & a_{62} & 0 & a_{64} & 0 & 0 & a_{67} & 0 \\ 0 & 0 & a_{73} & 0 & a_{75} & a_{76} & 0 & a_{78} \\ a_{81} & a_{82} & 0 & a_{84} & 0 & 0 & a_{87} & 0 \end{bmatrix}.$$

3.4 Solution of vector matrix differential equation

We use eigenvalue approach to solve the differential equation (3.11). The characteristic equation of matrix, A is given by

$$\lambda^8 - B_1\lambda^6 + B_2\lambda^4 - B_3\lambda^2 + B_4 = 0, \quad (3.12)$$

$$B_1 = a_{51} + a_{61} + a_{73} + a_{84} + a_{57}a_{75} + a_{67}a_{76}$$

$$B_2 = a_{51}a_{62} - a_{52}a_{61} + a_{51}a_{73} + a_{62}a_{73} + a_{51}a_{84} - a_{54}a_{81} + a_{62}a_{84} - a_{82}a_{64} + a_{73}a_{84} \\ - a_{54}a_{87}a_{75} + a_{51}a_{67}a_{76} - a_{52}a_{67}a_{75} + a_{87}a_{67}a_{75} + a_{87}a_{62}a_{78} + a_{57}a_{75}a_{84} - \\ a_{57}a_{78}a_{81} + a_{67}a_{76}a_{84} - a_{78}a_{82}a_{67}$$

$$B_3 = -a_{52}a_{61}a_{73} + a_{51}a_{62}a_{73} + a_{51}a_{62}a_{84} - a_{51}a_{62}a_{84} - a_{51}a_{64}a_{82} - a_{52}a_{61}a_{84} + \\ a_{52}a_{64}a_{81} + a_{54}a_{61}a_{82} + a_{51}a_{73}a_{84} - a_{54}a_{73}a_{81} + a_{62}a_{73}a_{84} - a_{64}a_{73}a_{82} - \\ a_{57}a_{61}a_{76}a_{84} + a_{57}a_{62}a_{75}a_{84} - a_{57}a_{64}a_{75}a_{82} + a_{57}a_{64}a_{76}a_{81} + a_{51}a_{67}a_{76}a_{84} - \\ 7a_{52}a_{67}a_{75}a_{84} + a_{54}a_{67}a_{75}a_{82} - a_{54}a_{67}a_{76}a_{81} + a_{57}a_{61}a_{78}a_{82} - a_{57}a_{62}a_{78}a_{81} - \\ a_{51}a_{67}a_{78}a_{82} + a_{52}a_{67}a_{78}a_{81} - a_{51}a_{87}a_{64}a_{76} + a_{52}a_{87}a_{64}a_{75} + a_{87}a_{54}a_{61}a_{75} - \\ a_{87}a_{54}a_{62}a_{75} + a_{51}a_{87}a_{62}a_{78} - a_{52}a_{87}a_{61}a_{78}$$

$$B_4 = a_{51}a_{62}a_{73}a_{84} - a_{51}a_{64}a_{73}a_{82} - a_{52}a_{61}a_{73}a_{84} + a_{52}a_{64}a_{73}a_{81} + a_{54}a_{61}a_{73}a_{82} - \\ a_{54}a_{62}a_{73}a_{81}$$

The roots of characteristic equation, $\pm\lambda_1$, $\pm\lambda_2$, $\pm\lambda_3$ and $\pm\lambda_4$ are eigenvalue of matrix A and their values are given by

$$\lambda_1 = \frac{B_1}{4} - \frac{1}{2}\beta_4 - \frac{1}{2}\beta_5 - \beta_6, \quad \lambda_2 = \frac{B_1}{4} - \frac{1}{2}\beta_4 + \frac{1}{2}\beta_5 - \beta_6, \\ \lambda_3 = \frac{B_1}{4} + \frac{1}{2}\beta_4 - \frac{1}{2}\beta_5 - \beta_6, \quad \lambda_4 = \frac{B_1}{4} + \frac{1}{2}\beta_4 - \frac{1}{2}\beta_5 + \beta_6.$$

where

$$\begin{aligned}
\beta_1 &= 2B_2^3 - 9B_1B_2B_3 + 27B_3^2 + 27B_1^2B_4 - 72B_2B_4, \quad \beta_6 = \sqrt{\frac{1}{\beta_3}(B_1^3 - 4B_1B_2 + 8B_3)}, \\
\beta_2 &= \sqrt{-4((B_2^2 - 3B_1B_3 + 12B_4)^3 + (2B_2^3 - 9B_1B_2B_3 + 27B_3^2 + 27B_1^2B_4 - 72B_2B_4)^2)^{\frac{1}{3}}}, \\
\beta_3 &= 4\sqrt{\frac{B_1^2}{4} - \frac{2B_2}{3} + \frac{2^{\frac{1}{3}}(B_2^2 - 3B_1B_3 + 12B_4)}{3(\beta_1 + \beta_2)} - \frac{1}{3 \times 2^{\frac{1}{3}}}(\beta_1 + \beta_2)}, \\
\beta_4 &= \sqrt{\left(\frac{B_1^2}{4} - \frac{2B_2}{3} + \frac{2^{\frac{1}{3}}}{3(\beta_1 + \beta_2)}(B_2^2 - 3B_1B_3 + 12B_4) + \frac{1}{3 \times 2^{\frac{1}{3}}}(\beta_1 + \beta_2)\right)}, \\
\beta_5 &= \sqrt{\left(\frac{B_1^2}{4} - \frac{4B_2}{3} - \frac{2^{\frac{1}{3}}}{3(\beta_1 + \beta_2)}(B_2^2 - 3B_1B_3 + 12B_4) + \frac{1}{3 \times 2^{\frac{1}{3}}}(\beta_1 + \beta_2)\right)}.
\end{aligned}$$

Thus, the full form of displacement components and temperature field in three dimensions are given by

$$(u, v, w, T) = \sum_{n=1}^4 (1, b_n, d_n, f_n) A_n e^{i(\alpha x + \beta y - \omega t)} e^{-\lambda_n z} \quad (3.13)$$

where b_n , d_n and f_n are coupling constants and their expression are given below

$$b_n = \frac{S_n}{Q_n}, \quad d_n = \frac{R_n}{Q_n}, \quad f_n = \frac{P_n}{Q_n}, \quad (n = 1, 2, 3, 4)$$

$$\begin{aligned}
R_n &= \frac{1}{a_0} \{P_n \lambda_n (-c_{41}^2 a_{52} + a_{54} a_{67}) + Q_n (a_{67} D_n \lambda_n + a_{52} \lambda_n q_3)\}, \quad a_0 = P_{12} a_{67} + E_n a_{52}, \\
S_n &= \frac{1}{a_{52}} \{P_{12} \lambda_n R_n - a_{54} P_n - D_n Q_n\}, \quad E_n = g_3 + (c_{31}^2 + 2c_{21}^2) \lambda_n - \iota (c_{21}^2 + c_{31}^2) \alpha \lambda_n, \\
D_n &= a_{51} + \lambda_n (\lambda_n - \iota c_{21} \beta), \quad P_{12} = \iota \alpha (c_{21}^2 - c_{31}^2), \quad g_3 = k_1^2 - (\alpha^2 + \beta^2), \\
F_n &= (c_{51}^2 a_{81} E_n - c_{51}^2 a_{67} a_{82} \lambda_n^2) (a_{67} D_n \lambda_n - a_{52} q_3), \quad P_n = F_n - Q_n, \\
Q_n &= (\lambda_n c_{51}^2 q_3 a_{81} - q_4 \lambda_n a_{67}) (P_{12} a_{67} \lambda_n^2 + E_n a_{52}), \quad Q_n = M_n - N_n, \\
M_n &= -\lambda_n \{ (a_{54} a_{67} \lambda_n + c_{41}^2 a_{52}) (E_n c_{51}^2 a_{81} c_{51}^2 a_{82} \lambda_n^2 a_{67}), \quad q_3 = -\iota \beta c_{21}^2, \\
N_n &= (c_{51}^2 a_{81} - F_n a_{67}) (P_{12} a_{67} \lambda_n^2 + E_n a_{52}), \quad q_4 = T_0 \omega \alpha c_{41}^2.
\end{aligned}$$

The stress tensors are given by

$$\begin{aligned}
\sigma_{xx} &= (\lambda + \mu) \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} + \lambda \frac{\partial w}{\partial z} - \gamma T, \quad \sigma_{yz} = \mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right), \\
&+ \frac{\partial w}{\partial x}), \quad \sigma_{zz} = \lambda \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} + (\lambda + \mu) \frac{\partial w}{\partial z} - \gamma T, \quad \sigma_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \\
\sigma_{zx} &= \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right), \quad \sigma_{yy} = \lambda \frac{\partial u}{\partial x} + (\lambda + \mu) \frac{\partial v}{\partial y} + \lambda \frac{\partial w}{\partial z} - \gamma T.
\end{aligned}$$

The stress components are obtained in the form of coupling parameter as

$$\begin{aligned}
\sigma_{xx} &= \sum_{n=1}^4 A_n \{ \iota \alpha (\mu + \lambda) + \iota \beta \lambda b_n - \lambda d_n \lambda_n - \gamma T \} e^{\iota(\alpha x + \beta y - \omega t)} e^{-\lambda_n z}, \\
\sigma_{yy} &= \sum_{n=1}^4 A_n \{ \iota \alpha \lambda + \iota \beta (\lambda + \mu) b_n - \lambda_n \lambda d_n - \gamma T \} e^{\iota(\alpha x + \beta y - \omega t)} e^{-\lambda_n z}, \\
\sigma_{zz} &= \sum_{n=1}^4 A_n \{ \lambda \iota (\alpha + \beta b_n) - (\lambda + \mu) d_n \lambda_n - \gamma T \} e^{\iota(\alpha x + \beta y - \omega t)} e^{-\lambda_n z}, \\
\sigma_{yx} &= \sum_{n=1}^4 \iota \mu A_n (\beta + \alpha b_n) e^{\iota(\alpha x + \beta y - \omega t)} e^{-\lambda_n z}, \\
\sigma_{zx} &= \sum_{n=1}^4 \mu A_n (\iota \alpha d_n - \lambda_n) e^{\iota(\alpha x + \beta y - \omega t)} e^{-\lambda_n z}, \\
\sigma_{zy} &= \sum_{n=1}^4 \mu A_n (\iota \beta d_n - \lambda_n b_n) e^{\iota(\alpha x + \beta y - \omega t)} e^{-\lambda_n z}.
\end{aligned}$$

3.5 Boundary conditions

The boundary surface at $z = 0$ is subjected to a time dependent thermal shock and the conditions are given by

$$T(x, y, 0, t) = F(t)H(a - |x|)(b - |y|), \quad (3.14)$$

$$\sigma_{zz}(x, y, 0, t) + \bar{\sigma}_{zz}(x, y, 0, t) = 0, \quad (3.15)$$

$$\sigma_{yz}(x, y, 0, t) + \bar{\sigma}_{yz}(x, y, 0, t) = 0, \quad (3.16)$$

$$\sigma_{xz}(x, y, 0, t) + \bar{\sigma}_{xz}(x, y, 0, t) = 0, \quad (3.17)$$

where H denotes Heaviside's function and $F(t) = \theta_0 e^{-\delta t}$ taking θ_0 as a constant.

Inserting the values of σ_{ij} , $\bar{\sigma}_{ij}$ and T from (1.28), (3.3), and (3.13) into Eqs.(3.14)-(3.17), we get the following matrix equation

$$bA = c, \quad (3.18)$$

where

$$\begin{aligned}
b_{1n} &= f_n, \quad b_{2n} = \iota \lambda (\alpha + b_n \beta) - (\lambda + 1) d_n \lambda_n - \gamma T + H_0^2 \{ (b_n - 1) \iota (\beta - \alpha) - 2 d_n \lambda_1 \}, \\
b_{3n} &= d_n \iota \beta - \lambda_n b_n, \quad b_{4n} = -\lambda_n + d_n \iota \alpha, \quad c = [\theta_0 e^{-(d+\omega)t + \iota(\alpha x + \beta y)}, 0, 0, 0]^T, \\
A &= [A_1, A_2, A_3, A_4]^T.
\end{aligned}$$

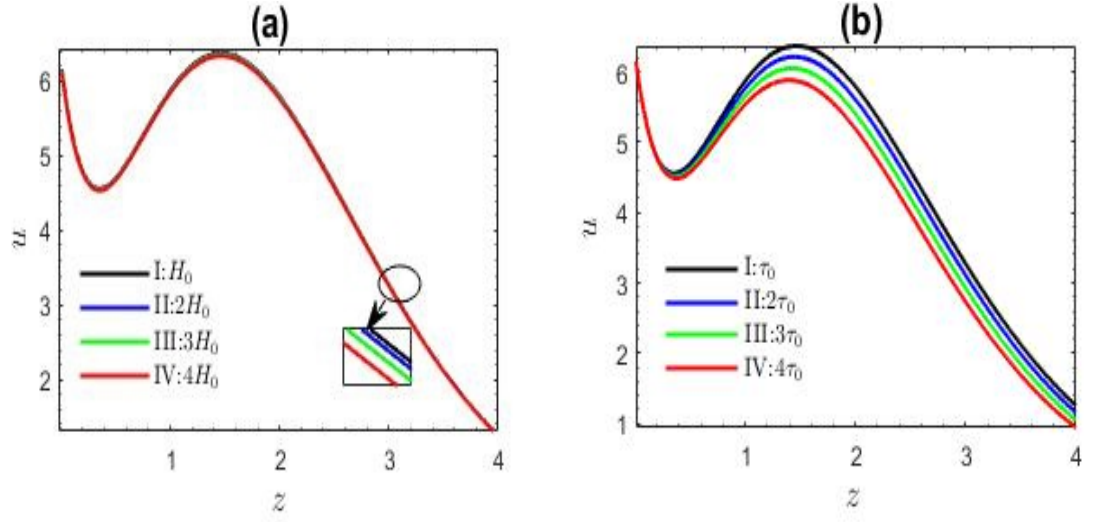
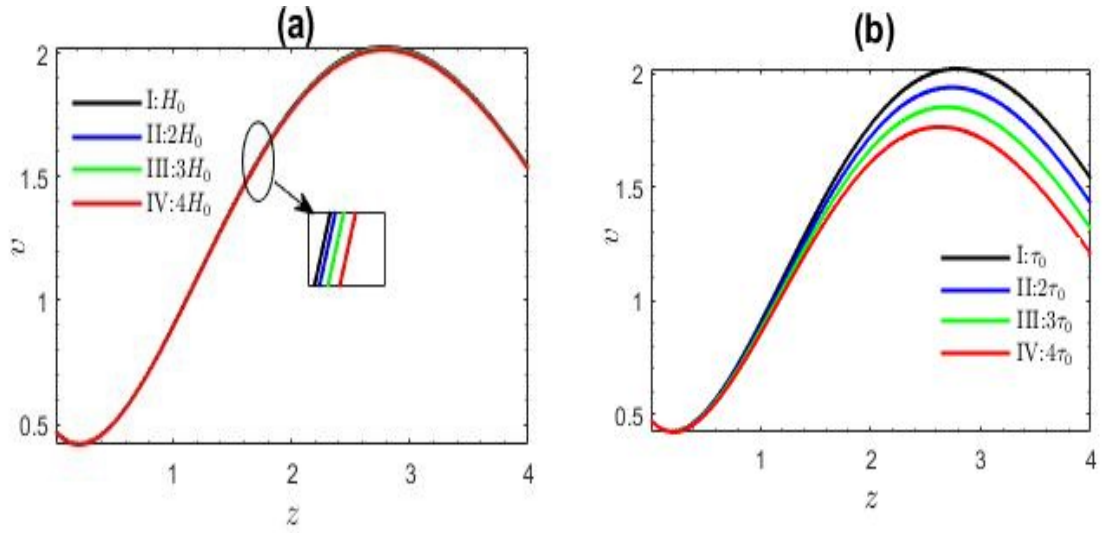
3.6 Numerical discussion

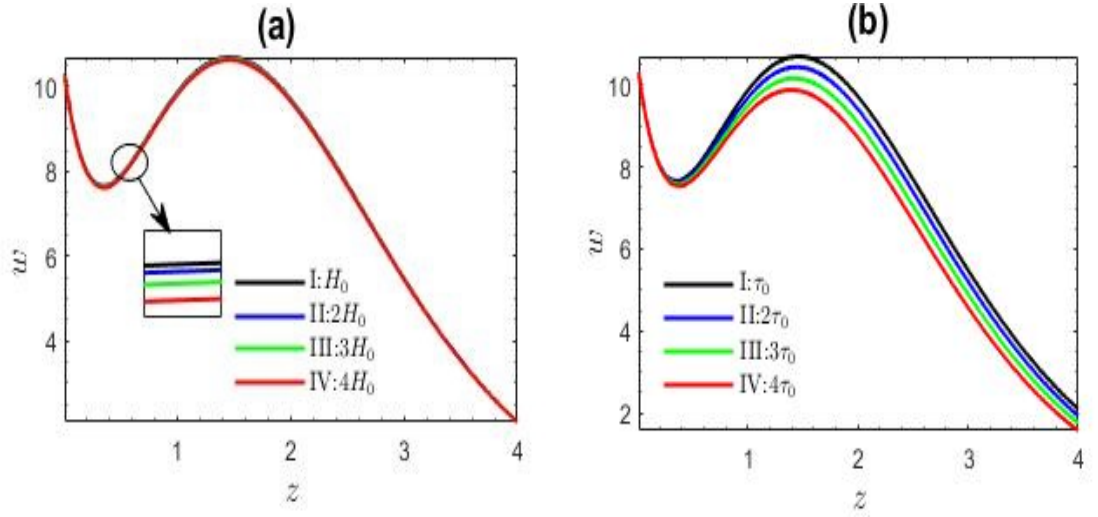
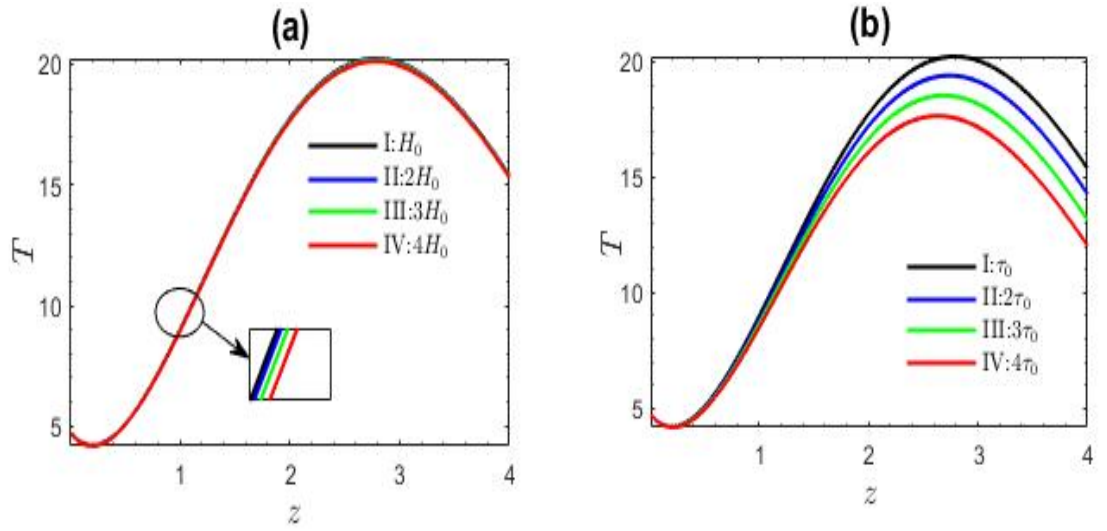
We have developed a Matlab program for (u, v, w, T) and stress tensors. The effect of H_0 and τ_0 on the propagation of waves have been examined. The following relevant values of parameters for the generalized thermoelastic solid are (Ezzat and Youssef, 2005)

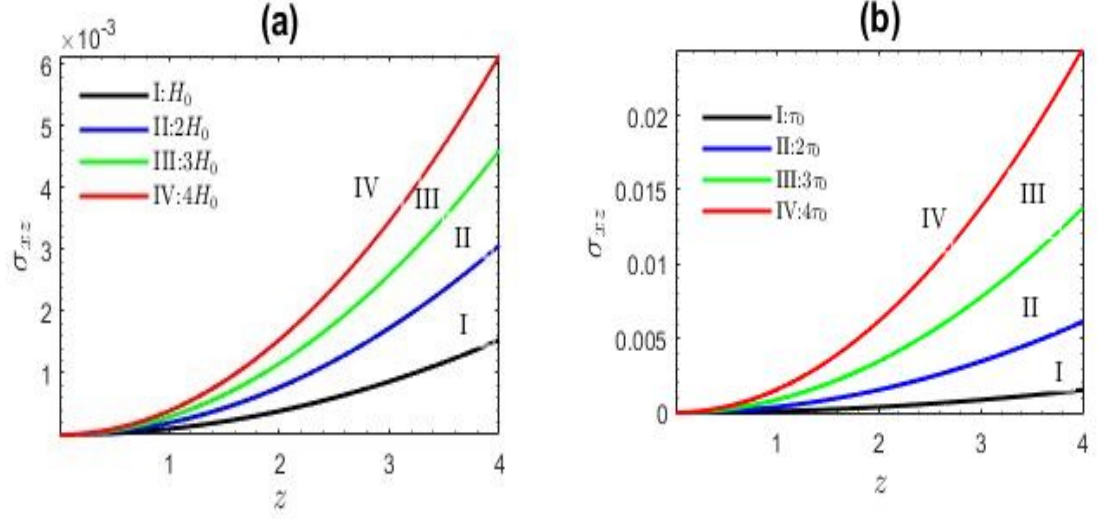
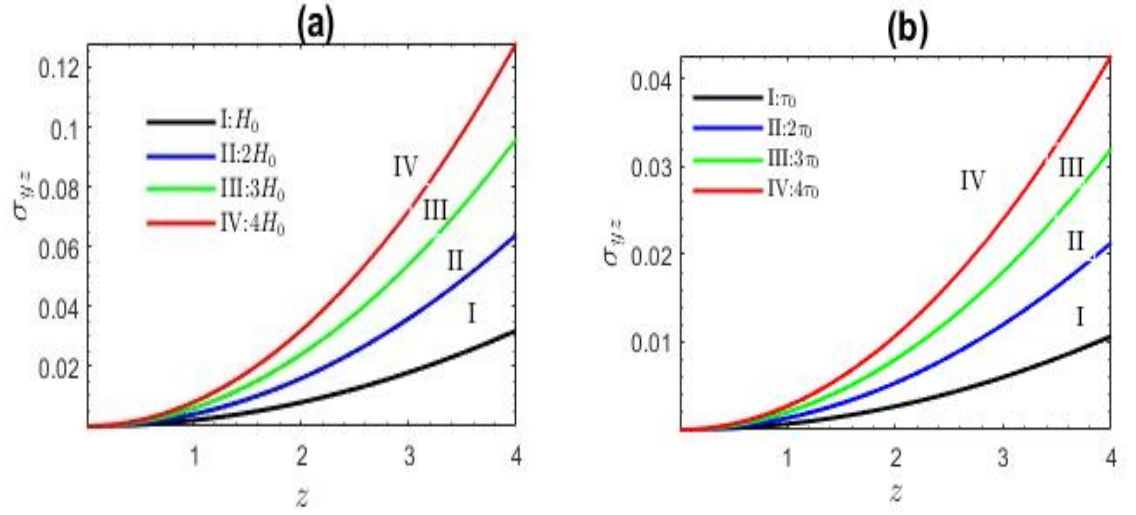
Parameters	Values	Units	Parameters	Values	Units
κ_0	386	$NK^{-1}s^{-1}$	α_t	1.78×10^{-5}	K^{-1}
C_e	383.1	$m^2K^{-1}s^{-2}$	ρ	8954	Kgm^{-3}
μ	3.86×10^{10}	Nm^{-2}	λ	7.76×10^{10}	Nm^{-2}
μ_0	$4\pi \times 10^{-7}$	Nms^2C^{-2}	H_0	10^3	$Cm^{-1}s^{-1}$
τ_0	0.02		T_0	293	K
θ_0	10		δ	0.1	
x	1		y	2	
α	1.2		β	1.4	
t	1				

Table 3.1: Values of the parameters

Figures 3.1 to 3.4 represent the variation of the displacement components and temperature field with z for different values of H_0 and τ_0 , while that of stress components are represented in Figures 3.5 to 3.7. We have seen a very interesting and beautiful effects of initial magnetic field and relaxation time on $u, v, w, T, \sigma_{xz}, \sigma_{yz}$ and σ_{zz} . In Figure 3.1, the displacement component u starts from certain value which decreases with the increase of z upto $z = 0.29$ and then it increases and decreases with the increase of z . We have seen the effect of H_0 is smaller than τ_0 . It is also clearly observed that the values of u decrease with the increase of H_0 and τ_0 . But the effect of τ_0 is very small near the origin. Figure 3.2 shows that values of v increase upto $z = 2.67$ and then decrease with the increase of z . It is observed that the values of v decrease with the increase of H_0 and τ_0 . Figure 3.3 shows similar nature of w with u . The variation of T with z are shown in Figure 3.4 which is similar with the nature of v .

Figure 3.1: Variation of u for different values of (a) H_0 and (b) τ_0 Figure 3.2: Variation of v for different values of (a) H_0 and (b) τ_0

Figure 3.3: Variation of w for different values of (a) H_0 and (b) τ_0 Figure 3.4: Variation of T for different values of (a) H_0 and (b) τ_0

Figure 3.5: Variation of σ_{xz} for different values of (a) H_0 and (b) τ_0 Figure 3.6: Variation of σ_{yz} for different values of (a) H_0 and (b) τ_0

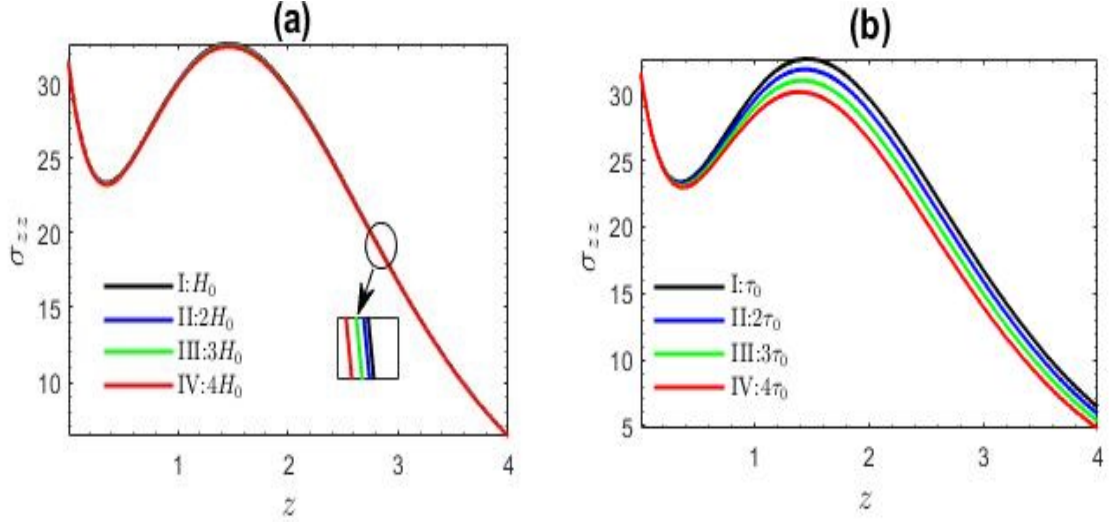


Figure 3.7: Variation of σ_{zz} for different values of (a) H_0 and (b) τ_0

The variation of σ_{xz} and σ_{yz} with z are shown in Figures 3.5 and 3.6 respectively. It is observed that both these tensors increase with the increase of z at different values of H_0 and τ_0 . The values of these tensors increase with the increase H_0 and τ_0 . Similar natures of σ_{zz} with u are observed in Figure 3.7. Thus, we have seen that all the effects of H_0 and τ_0 to the displacement and stress are very small near origin.

3.7 Conclusions

We have analysed the 3D problem of wave propagation in generalized magneto thermoelastic medium. The eigenvalue technique has been used to find the expressions of displacement components and stress tensors. All these expressions are computed numerically for a particular model. The effects of H_0 and τ_0 on displacement, stress tensors and temperature field are depicted graphically. We can conclude the following points:

- The effect τ_0 on the displacement and stress is very small near origin. The effect of H_0 on the components of displacement and temperature field is also very small.
- The values of σ_{xz} and σ_{yz} increase with the increase of z .

- The values of σ_{xz} and σ_{yz} increase with the increase of H_0 and τ_0 .
- The values of u, v, w and σ_{zz} decrease with the increase of H_0 and τ_0 .
- Similar natures are observed for u, w and σ_{zz} .
- Similar natures are observed for v and T .

Chapter 4

Elastic wave from a half-space of micropolar generalized magneto-thermoelastic materials

4.1 Introduction

The theory of micropolar elasticity, which accounts for materials with microstructural characteristics, has gained considerable importance in recent decades. This theory has been particularly applied to problems involving deformation in granular media and complex multimolecular systems. Eringen and Shuhubi(1964) presented the basic field equations, boundary conditions and constitutive equations of simple micro-elastic solids. Eringen(1966) gave the equations of motion, constitutive equations and boundary conditions for micropolar solids.

In the present work, the problem of reflection of homogeneous plane waves from the free surface of generalized magneto micropolar thermoelastic half-space using modified Ohm's law and generalized Fourier law. There are five waves propagated in the medium and these waves are longitudinal, transverse, micropolar, thermal and another wave due to magnetic field. The phase velocities of these waves are obtained analytically and numerically. Using appropriate boundary conditions, the amplitude and energy ratios corresponding to the reflected waves are obtained analytically and numerically. Magnetic and thermal effects on the reflected waves are investigated and found the conservation law of energy in all cases.

4.2 Basic Equations

The linear momentum equation in the absence of body force that accounts for Lorentz force can be expressed in the following way

$$\sigma_{ji,j} + (\mathbf{J} \times \mathbf{B})_i + \rho_e E_i = \rho \ddot{u}_i. \quad (4.1)$$

The angular momentum equation of balance with electromagnetic couples excluding body couples can be given by

$$e_{ijk}\sigma_{jk} + \mu_{ji,j} + e_{ijk}x_j(\mathbf{J} \times \mathbf{B})_k = \rho j w_i. \quad (4.2)$$

The linear form of Equations (1.26), (4.1), (4.2) are re-written as (1.24) and

$$\sigma_{ji,j} + \mu_0(\mathbf{J} \times \mathbf{H}_0)_i = \rho \ddot{u}_i, \quad e_{ijk}\sigma_{jk} + \mu_{ji,j} = \rho j \ddot{w}_i. \quad (4.3)$$

The equation of heat conduction for the linear theory may be given as

$$\kappa \nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (\rho C_e T + \gamma T_0 \Sigma) + \pi_0 \nabla \cdot \mathbf{J}. \quad (4.4)$$

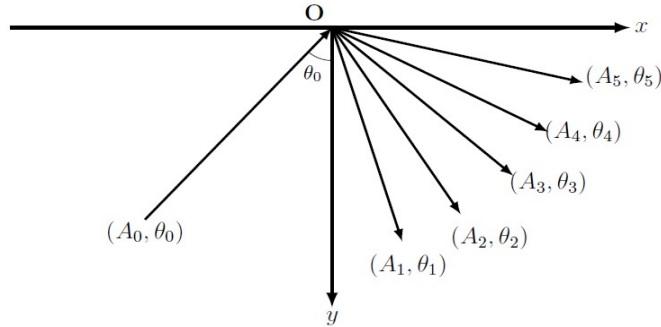


Figure 4.1: Geometry of the Problem

4.3 Formulation of the problem

We consider the two-dimensional problem of wave propagation in the xy plane for the Cartesian coordinate axis such that y -axis lies vertically with pointing downward and x -axis horizontally as shown in Figure 4.1. Assume that a constant magnetic field exists in the homogeneous micropolar thermoelastic material with a fixed direction,

$\mathbf{H}_0 = (0, 0, H_0)$ and the induced magnetic field, $\mathbf{h} = (0, 0, h)$. The components of displacement, the micro-rotation, the electric field normal to magnetic field and the electric current density parallel to the electric field are given by

$$\mathbf{u} = (u, v, 0), \quad \mathbf{w} = (0, 0, w), \quad \mathbf{E} = (E_1, E_2, 0), \quad \mathbf{J} = (J_1, J_2, 0), \quad \frac{\partial}{\partial z} \equiv 0.$$

We obtain the following sets of equations from Eq. (1.21) and (1.24):

$$\begin{aligned} \frac{\partial h}{\partial y} &= \sigma_0 \left(E_1 + \mu_0 H_0 \frac{\partial v}{\partial t} \right) + \epsilon_0 \frac{\partial E_1}{\partial t} - \kappa_0 \frac{\partial T}{\partial x}, \\ \frac{\partial h}{\partial x} &= -\sigma_0 \left(E_2 - \mu_0 H_0 \frac{\partial u}{\partial t} \right) - \epsilon_0 \frac{\partial E_2}{\partial t} + \kappa_0 \frac{\partial T}{\partial y}, \end{aligned} \quad (4.5)$$

$$\begin{aligned} \frac{\partial E_1}{\partial y} - \frac{\partial E_2}{\partial x} &= \mu_0 \frac{\partial h}{\partial t}, \\ J_1 &= \sigma_0 \left(E_1 + \mu_0 H_0 \frac{\partial v}{\partial t} \right) - \kappa_0 \frac{\partial T}{\partial x}, \\ J_2 &= \sigma_0 \left(E_2 - \mu_0 H_0 \frac{\partial u}{\partial t} \right) - \kappa_0 \frac{\partial T}{\partial y}. \end{aligned} \quad (4.6)$$

Using Equations (1.30) and (1.31) into (1.24) and (4.3), we get

$$(\mu + \alpha) \nabla^2 \mathbf{u} + (\lambda + \mu - \alpha) \nabla (\nabla \cdot \mathbf{u}) + 2\alpha \nabla \times \mathbf{w} - \gamma \nabla T + \mu_0 (\mathbf{J} \times \mathbf{H}_0) = \rho \ddot{\mathbf{u}}, \quad (4.7)$$

$$(\nu + \beta) \nabla^2 \mathbf{w} + (\epsilon + \nu - \beta) \nabla (\nabla \cdot \mathbf{w}) + 2\alpha \nabla \times \mathbf{u} - 4\alpha \mathbf{w} = \rho j \ddot{\mathbf{w}}. \quad (4.8)$$

We introduce the following parameters for making non-dimensional system:

$$\begin{aligned} x^* &= c_0 \eta_0 x, \quad y^* = c_0 \eta_0 y, \quad u^* = c_0 \eta_0 u, \quad v^* = c_0 \eta_0 v, \quad t^* = c_0^2 \eta_0 t, \quad \tau_0^* = c_0^2 \eta_0 \tau_0, \\ T^* &= \frac{\gamma T}{\rho c_0^2}, \quad w^* = \frac{\alpha w}{\mu + \alpha}, \quad \sigma_{ij}^* = \frac{\sigma_{ij}}{\mu + \alpha}, \quad \mu_{ij}^* = \frac{\alpha \mu_{ij}}{c_0 \eta_0 (\mu + \alpha) (\nu + \beta)}, \\ h^* &= \frac{\eta_0 h}{\sigma_0 \mu_0 H_0}, \quad E_i^* = \frac{\eta_0 E_i}{\sigma_0 \mu_0^2 H_0 c_0}, \quad J_i^* = \frac{\eta_0 J_i}{\sigma_0^2 \mu_0^2 H_0 c_0}, \quad (i = 1, 2), \quad c_0^2 = \frac{\lambda + 2\mu}{\rho}, \quad \eta_0 = \frac{\rho C_e}{\kappa_0}. \end{aligned}$$

Now, the new non-dimensional equations of motion are given by

$$\beta_1^2 \frac{\partial e}{\partial x} + \frac{\partial \phi}{\partial y} + 2 \frac{\partial w}{\partial y} + \beta_1^2 \nu_1^2 \epsilon_2 E_2 - \beta_1^2 \left(\frac{\partial T}{\partial x} + \kappa_1 \frac{\partial T}{\partial y} \right) = \beta_1^2 \left(\frac{\partial^2 u}{\partial t^2} + \nu_1 \epsilon_2 \frac{\partial u}{\partial t} \right), \quad (4.9)$$

$$\beta_1^2 \frac{\partial e}{\partial y} - \frac{\partial \phi}{\partial x} - 2 \frac{\partial w}{\partial x} - \beta_1^2 \nu_1^2 \epsilon_2 E_1 - \beta_1^2 \left(\frac{\partial T}{\partial y} - \kappa_1 \frac{\partial T}{\partial x} \right) = \beta_1^2 \left(\frac{\partial^2 v}{\partial t^2} + \nu_1 \epsilon_2 \frac{\partial v}{\partial t} \right), \quad (4.10)$$

$$\nabla^2 w - g_1 \phi - g_2 w = g_3 \frac{\partial^2 w}{\partial t^2}, \quad (4.11)$$

$$\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (T + \epsilon_1 \nabla \cdot \mathbf{u}) - \pi_1 \frac{\partial}{\partial t} \left(\frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y} \right). \quad (4.12)$$

Thus, the non-dimensional expression of stress tensors, components of electric and magnetic fields are given by

$$\begin{aligned}\sigma_{xx} &= \beta_1^2 \left(\frac{\partial u}{\partial x} - T \right) + \delta_6 \frac{\partial v}{\partial y}, \quad \sigma_{yy} = \delta_6 \frac{\partial u}{\partial x} + \beta_1^2 \left(\frac{\partial v}{\partial y} - T \right), \quad \sigma_{xy} = \frac{\partial v}{\partial x} + \delta_7 \frac{\partial u}{\partial y} - 2w, \\ \sigma_{yx} &= \frac{\partial u}{\partial y} + \delta_7 \frac{\partial v}{\partial x} + 2w, \quad \mu_{xz} = \frac{\partial w}{\partial x}, \quad \mu_{zx} = \delta_8 \frac{\partial w}{\partial x}, \quad \mu_{yz} = \frac{\partial w}{\partial y}, \quad \mu_{zy} = \delta_8 \frac{\partial w}{\partial y}, \\ \frac{\partial h}{\partial y} &= \nu_1 E_1 + \frac{\partial v}{\partial t} + V^2 \frac{\partial E_1}{\partial t} - \kappa_2 \frac{\partial T}{\partial x}, \quad \frac{\partial h}{\partial x} = -\nu_1 E_2 + \frac{\partial u}{\partial t} - V^2 \frac{\partial E_2}{\partial t} + \kappa_2 \frac{\partial T}{\partial y}, \\ \frac{\partial E_1}{\partial y} - \frac{\partial E_2}{\partial x} &= \frac{\partial h}{\partial t}, \quad J_1 = E_1 + \frac{1}{\nu_1} \frac{\partial v}{\partial t} - \kappa_3 \frac{\partial T}{\partial x}, \quad J_2 = E_2 - \frac{1}{\nu_1} \frac{\partial u}{\partial t} - \kappa_3 \frac{\partial T}{\partial y},\end{aligned}$$

where

$$\begin{aligned}\beta_1^2 &= \frac{\lambda + 2\mu}{\mu + \alpha}, \quad \nu_1 = \frac{\sigma_0 \mu_0}{\eta_0}, \quad V^2 = \frac{c_0^2}{c^2}, \quad c^2 = \frac{1}{\mu_0 \epsilon_0}, \quad \epsilon_1 = \frac{\gamma^2 T_0}{\rho c_0^2 \eta_0 \kappa}, \quad \epsilon_2 = \frac{\mu_0 H_0^2}{\rho c_0^2}, \\ \kappa_1 &= \frac{\mu_0 H_0 \kappa_0}{\gamma}, \quad \kappa_2 = \frac{\kappa_1}{\nu_1 \epsilon_2}, \quad \kappa_3 = \frac{\kappa_2}{\nu_1}, \quad g_1 = \frac{2\alpha^2}{c_0^2 \eta_0^2 (\nu + \beta)(\mu + \alpha)}, \quad g_2 = \frac{4\alpha}{c_0^2 \eta_0^2 (\nu + \beta)}, \\ g_3 &= \frac{\rho j c_0^2}{\nu + \beta}, \quad \pi_1 = \pi_0^* \epsilon_2 V^2, \quad \pi_0^* = \frac{\pi_0 \sigma_0 \gamma}{\kappa H_0}, \quad \delta_6 = \frac{\lambda}{\mu + \alpha}, \quad \delta_7 = \frac{\mu - \alpha}{\mu + \alpha}, \quad \delta_8 = \frac{\nu - \beta}{\nu + \beta}, \\ \nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \quad \phi = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}.\end{aligned}$$

Using above expressions into the Equations (4.9)-(4.12) we get

$$(\nabla^2 - \frac{\partial^2}{\partial t^2} - \nu_1^2 \epsilon_2 \frac{\partial}{\partial t})e = \nabla^2 T + \nu_1^2 \epsilon_2 \frac{\partial h}{\partial t}, \quad (4.13)$$

$$\left[(1 - \epsilon_3) \nabla^2 - \beta_1^2 \left(\frac{\partial^2}{\partial t^2} + \nu_1 \epsilon_2 \frac{\partial}{\partial t} \right) \right] \phi = (\beta_1^2 \kappa_1 - \epsilon_{30}) \nabla^2 T - 2 \nabla^2 w, \quad (4.14)$$

$$(\nabla^2 - V^2 \frac{\partial^2}{\partial t^2} + \nu_1 \frac{\partial}{\partial t})h = \frac{\partial e}{\partial t}, \quad (4.15)$$

$$\left[(\nabla^2 - V^2 \frac{\partial}{\partial t}) \frac{\iota s}{\nu_1 + \iota s V^2} + \frac{\partial}{\partial t} \right] \phi = \left[(\nabla^2 - V^2 \frac{\partial}{\partial t}) \frac{\kappa_2}{\nu_1 + \iota s V^2} - \kappa_2 \right] \nabla^2 T, \quad (4.16)$$

$$\left[(1 + \pi_2) \nabla^2 - \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \right] T = \epsilon_1 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) e + \pi_{20} \frac{\partial \phi}{\partial t}, \quad (4.17)$$

where

$$\epsilon_3 = \frac{\beta_1^2 \epsilon_2}{\nu_1 + \iota s V^2}, \quad \epsilon_{30} = \frac{\beta_1^2 \nu_1^2 \epsilon_2 \kappa_2}{\nu_1 + \iota s V^2}, \quad \pi_2 = \frac{\kappa_2 \pi_1}{\nu_1 + \iota s V^2}, \quad \pi_{20} = \frac{\iota s \pi_1}{\nu_1 + \iota s V^2}.$$

4.4 Wave propagation

The following variables can be expressed in the form of plane wave by expressing in the exponential form as (Ezzat and Awad, 2009)

$$(e, \phi, u, v, w, T, h) = (A_0, B_0, C_0, D_0, E_0, F_0, G_0) \exp\{i(st - k(xp_1 + yp_2))\}, \quad (4.18)$$

where $p = (p_1, p_2, 0) = (\sin \theta, \cos \theta, 0)$ is the propagation vector and $s = kv$ is angular velocity and k is the wavenumber.

Using Eq. (4.18) into (4.13)-(4.17), we get

$$g_1 B_0 + (k^2 - g_4) E_0 = 0, \quad (g_5 - k^2) A_0 - k^2 F_0 + \iota \nu_1^2 \epsilon_2 s G_0 = 0, \quad (4.19)$$

$$\iota s A_0 + (k^2 - g_6) G_0 = 0, \quad g_9 \epsilon_1 A_0 - \iota g_7 s B_0 + (g_8 + k^2 \kappa_2 g_4 - k^2) F_0 = 0, \quad (4.20)$$

$$(g_{12} - k^2) B_0 - 2k^2 E_0 + k^2 g_{11} F_0 = 0. \quad (4.21)$$

The non-singular solutions of Eqs. (4.13)-(4.17) give

$$k^{10} - k^8 r_1 + k^6 r_2 - k^4 r_3 + k^2 r_4 - r_5 = 0, \quad (4.22)$$

where

$$\begin{aligned} g_4 &= -g_2 + s^2 g_3, \quad g_5 = s^2 - \nu_1 \epsilon_2 \iota s, \quad g_6 = V^2 s^2 + \nu_1 \iota s, \quad g_7 = \frac{\iota s}{\nu_1 + V^2 \iota s}, \\ g_8 &= -\iota s + \tau_0 s^2, \quad g_{10} = \frac{-\beta_1^2 \nu_1 \epsilon_2}{\nu_1 + V^2 \iota s}, \quad g_{11} = \beta_1^2 \kappa_1 + g_{10} \kappa_2, \quad g_{12} = -\beta_1^2 (-s^2 + \nu_1 \epsilon_2 \iota s), \\ m_{14} &= -(2m_4 g_1 + m_9), \quad m_3 = 2g_5 g_8, \quad m_5 = -g_{11} g_7 \iota s + g_{12} - \kappa_2 g_4 g_{12} + \kappa_2 g_4, \\ m_{12} &= -g_5 g_8 g_{11} m_8 + m_6 g_6 g_4, \quad r_4 = \frac{-m_{17}}{m_4}, \quad m_6 = -m_5 + m_4 g_5 - g_6 \epsilon_1, \quad m_8 = -(g_6 + g_4), \\ m_9 &= m_6 - m_8 m_4, \quad m_{10} = m_7 + m_6 m_8 - m_4 g_6 g_4, \quad m_{11} = -g_5 g_8 g_{11} + m_7 m_8 + m_6 g_6 g_4, \\ m_7 &= g_8 g_{11} + m_5 g_5 + g_{12} g_6 \epsilon_1, \quad m_{13} = -g_6 g_4 g_5 g_8 g_{11}, \quad m_{15} = -2m_4 g_1 g_6 - m_2 g_1 \\ &\quad - s^2 g_7 g_{11} + m_{10}, \quad g_9 = -(\iota s + \tau_0 s^2), \quad m_{18} = \iota s^3 g_4 g_{12} g_7 \epsilon_2 \nu_1^2 + m_{13}, \\ m_2 &= 2g_6 \epsilon_1 + 2\kappa_2 g_5 g_4 - 2(g_5 + g_8), \quad r_1 = \frac{m_{14}}{m_4}, \quad r_2 = \frac{-m_{15}}{m_4}, \quad r_3 = \frac{m_{16}}{m_4}, \\ m_4 &= -1 + \kappa_2 g_4, \quad m_{16} = -2s^2 \nu_1^2 \epsilon_2 g_1 - 2\kappa_2 g_4 g_1 s^2 \nu_1^2 \epsilon_2 + m_2 g_6 g_1 - m_3 g_1 \\ &\quad + g_7 g_{11} s^2 g_4 - \iota s^3 \nu_1^2 \epsilon_2 g_7 + m_{11}, \quad m_{17} = -2s^2 \nu_1^2 \epsilon_2 g_1 g_8 + m_3 g_6 g_1 + \iota s^3 g_4 \nu_1^2 \epsilon_2 g_7 \\ &\quad + \iota s^3 g_{12} g_7 + s^3 \iota g_{12} g_7 \epsilon_2 \nu_1^2 + m_{12}, \quad r_5 = \frac{m_{18}}{m_4}. \end{aligned}$$

This equation gives the wavenumbers and corresponding phase velocities of the five coupled body waves propagating in the micropolar generalized magneto-thermoelastic

materials.

Thus, the expression of variables corresponding to components of micro-rotation and displacement, temperature, and magnetic field can be written as

$$(e, \phi, u, v, w, T, h) = \sum_{i=0}^5 (1, \alpha_i, \Omega_i, \delta_i, \eta_i, \gamma_i, \mu_i) A_i \exp\{\iota \left(st - k_i(xp_1^{(i)} + yp_2^{(i)}) \right)\}, \quad (4.23)$$

where A_i is the constant amplitude, $(p_1^{(0)}, p_2^{(0)}) = (\sin \theta_0, \cos \theta_0)$ and $(p_1^{(i)}, p_2^{(i)}) = (\sin \theta_i, -\cos \theta_i)$ are the propagation vectors of the incident and reflected waves. The expression of the coupling parameters, α_i , η_i , γ_i , and μ_i is given below.

$$\begin{aligned} \mu_i &= \frac{\alpha_i \iota s}{g_6 - k_i^2}, \gamma_i = \frac{(-k_i^2 + g_6)(k_i^2 - g_5) + \nu_1^2 \epsilon_2 s^2}{(g_6 - k_i^2)k_i^2}, \eta_i = \frac{\alpha g_1 \alpha_i}{g_4 - k_i^2}, \\ \alpha_i &= \frac{1}{g_7 \iota s} \{(-k_i^2 + g_8 + k_i^2 \kappa_2 g_4) \gamma_i + g_9 \epsilon_1\}. \end{aligned}$$

The expressions, Ω_i and δ_i are obtained from the relations: $e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ and $\phi = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$ as

$$\Omega_i = \frac{\alpha_i - \iota k_i p_1^{(i)} \delta_i}{k_i p_2^{(i)}}, \quad \delta_i = \iota \frac{(p_2^{(i)} - \alpha_i p_1^{(i)})}{p_1^{(i)2} - k_i p_2^{(i)2}}.$$

The Snell's law, for the problem, is given by

$$k_0 \sin \theta_0 = k_i \sin \theta_i, \quad (i = 1, 2, 3, 4, 5). \quad (4.24)$$

4.5 Appropriate Boundary Conditions

The free surface of the continuum body of magneto micropolar thermoelastic material is observed as the point where the boundary conditions are defined. These conditions, at $y = 0$, are given by

$$\begin{aligned} \sigma_{xx}(x, 0, t) &= 0, \quad \sigma_{xy}(x, 0, t) = 0, \quad \mu_{xz}(x, 0, t) = 0, \\ T(x, 0, t) &= 0, \quad h(x, 0, t) = 0, \end{aligned} \quad (4.25)$$

where

$$\sigma_{xx} = \beta_1^2 \left(\frac{\partial u}{\partial x} - T \right) + \delta_6 \frac{\partial v}{\partial y}, \quad \sigma_{xy} = \frac{\partial v}{\partial x} + \delta_7 \frac{\partial u}{\partial y} - 2w, \quad \mu_{xz} = \frac{\partial w}{\partial x}.$$

Using Eqs.(4.23) and (4.24) into (4.25), we get a matrix equation of the form

$$[a_{ij}][Z] = [N], \quad (i, j = 1, 2, 3, 4, 5) \quad (4.26)$$

where a_{ij} is 5×5 matrix, $Z = [Z_1, Z_2, Z_3, Z_4, Z_5]^t$ is the amplitude ratio with $Z_i = \frac{A_i}{A_0}$, $N = [N_1, N_2, N_3, N_4, N_5]^t$ and their expressions are given below:

$$\begin{aligned} a_{1i} &= \iota\{(\mu_0 H_0^2 + \beta_1^2 - 2)k_0 p_1^{(0)} - (\mu_0 H_0^2 + \beta_1^2)\Omega_i k_i p_2^{(i)}\} + \beta_1^2 \gamma_i, \\ a_{2i} &= k_i p_2^{(i)} - \Omega_i k_0 p_1^{(0)} + 2\Omega_i, \quad a_{3i} = \eta_i, \quad a_{4i} = k_i p_2^i \gamma_i, \quad a_{5i} = \mu_i, \\ N_1 &= \iota\{(\mu_0 H_0^2 + \beta_1^2 - 2)k_0 p_1^{(0)} + (\mu_0 H_0^2 + \beta_1^2)\Omega_i k_0 p_2^{(0)}\} - \beta_1^2 \gamma_0, \\ N_2 &= k_0(p_2^{(0)} - \Omega_0 p_1^{(0)}) + 2\Omega_0, \quad N_3 = \eta_0, \quad N_4 = \gamma_0 k_0 p_2^{(0)}, \quad N_5 = -\mu_0. \end{aligned}$$

It may be noted that amplitude ratios depend on angle of propagation, elastic constants, micropolar, thermal and coupling parameters.

4.6 Energy Partition

The incident energy is distributed among the reflected waves at the boundary surface. The power per unit area represents the energy flux across the surface element and is given by (Tomar and Gogna, 1995; Tung, 2021; Khan et al., 2025)

$$\mathcal{P} = \langle \sigma_{xx}^i, \dot{u}^j \rangle + \langle \sigma_{xy}^i, \dot{v}^j \rangle + \langle \mu_{ij}^i, w^j \rangle, \quad (i, j = 0, 1, 2, 3, 4, 5), \quad (4.27)$$

where $i, j = 0$ represent the incident wave, while $i, j = 1, 2, 3, 4, 5$ represent the reflected waves. The energy distribution of the incident and reflected waves are given by

$$E_{inc} = A_0^2 s \{ p_2^{(0)} k_0 \Omega_0 \delta_0 (\delta_6 + \delta_7) + k_0 p_1^{(0)} (\beta_1^2 \Omega_0^2 + \delta_0^2 - \eta_0^2) + \iota (\beta_1^2 \gamma_0 - 2\eta_0 \delta_0) \}, \quad (4.28)$$

$$E_r = A_i^2 s \{ p_2^{(i)} k_i \Omega_i \delta_i (\delta_6 + \delta_7) + k_0 p_1^{(0)} (\beta_1^2 \Omega_i^2 + \delta_i^2 - \eta_i^2) + \iota (\beta_1^2 \gamma_i - 2\eta_i \delta_i) \} \quad (4.29)$$

Thus, the energy ratios corresponding to reflected waves, E_i ($i = 1, 2, 3, 4, 5$) are given by

$$E_i = \frac{p_2^{(i)} k_i b_{ri} + p_1^{(0)} k_0 d_{ri} + e_{ri}}{p_2^{(0)} k_0 b_{r0} + p_1^{(0)} k_0 d_{r0} + e_{r0}} Z_i^2, \quad (4.30)$$

where

$$b_{r0} = \Omega_0 \delta_0 (\delta_6 + \delta_7), \quad d_{r0} = \beta_1^2 \Omega_0^2 + \delta_0^2 - \eta_0^2, \quad e_{r0} = \iota(\beta_1^2 \gamma_0 - 2\eta_0 \delta_0),$$

$$b_{ri} = \Omega_i \delta_i (\delta_6 + \delta_7), \quad d_{ri} = \beta_1^2 \Omega_i^2 + \delta_i^2 - \eta_i^2, \quad e_{ri} = \iota(\beta_1^2 \gamma_i - 2\eta_i \delta_i).$$

The sum of energy ratios at the boundary surface for the five reflected waves is equal to unity. This shows the conservation law of energy for this specific problem.

4.7 Special cases

Case 1: If we neglect micropolar effect, the problem reduces to the reflection of waves from a plane free half-space of generalized magneto thermoelastic material. In this case, $\alpha = \beta = \epsilon = \nu = j = 0$ and the amplitude are given by Eqs.(4.26) and (4.30) with the following modified values

$$a_{3i} = a_{5i} = N_3 = N_5 = 0, \quad a_{2i} = k_i p_2^{(i)} - \Omega_i k_0 p_1^{(0)}, \quad N_2 = k_0(p_2^{(0)} + \Omega_0 p_1^{(0)}).$$

These results are perfectly matched with those of Singh and Boruah(2023) given in Chapter 2.

Case 2: If we neglect the magnetic effect, the problem reduces to the problem of reflection of waves from the half-space of generalized thermoelastic material. In this case, $H_0 = h = 0$ and the amplitude are given by with the Eqs.(4.26) and (4.30) following modified values

$$a_{1i} = \iota\{(\beta_1^2 - 2)k_0 p_1^{(0)} - \beta_1^2 \Omega_i k_i p_2^{(i)}\} + \beta_1^2 \gamma_i,$$

$$N_1 = \iota\{(\beta_1^2 - 2)k_0 p_1^{(0)} + \beta_1^2 \Omega_i k_0 p_2^{(0)}\} - \beta_1^2 \gamma_0,$$

These results are perfectly matched with the special case 1 of Singh and Boruah(2023), which are in Chapter 2.

Case 3: If we neglect thermal effect, the problem reduces to the problem of reflection of waves from the elastic half-space. In this case, $\kappa = C_e = \epsilon_1 = \gamma = \alpha_T = 0$ and the amplitude and energy ratios are respectively given by Eqs.(4.26) and (4.30) with the following modified values The amplitude and energy ratios are respectively given by

$$Z_1 = \frac{\sin 2\theta_0 \sin 2\theta_2 - \beta_1^2 \cos^2 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \beta_1^2 \cos^2 2\theta_2}, \quad Z_2 = \frac{2\beta_1 \sin 2\theta_0 \cos 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \beta_1^2 \cos^2 2\theta_2}, \quad (4.31)$$

$$E_1 = Z_1^2, \quad E_2 = \frac{\cos \theta_2 Z_2^2}{\beta_1^2 \cos \theta_0}. \quad (4.32)$$

These results exactly match with those of Achenbach(1976) for classical elasticity.

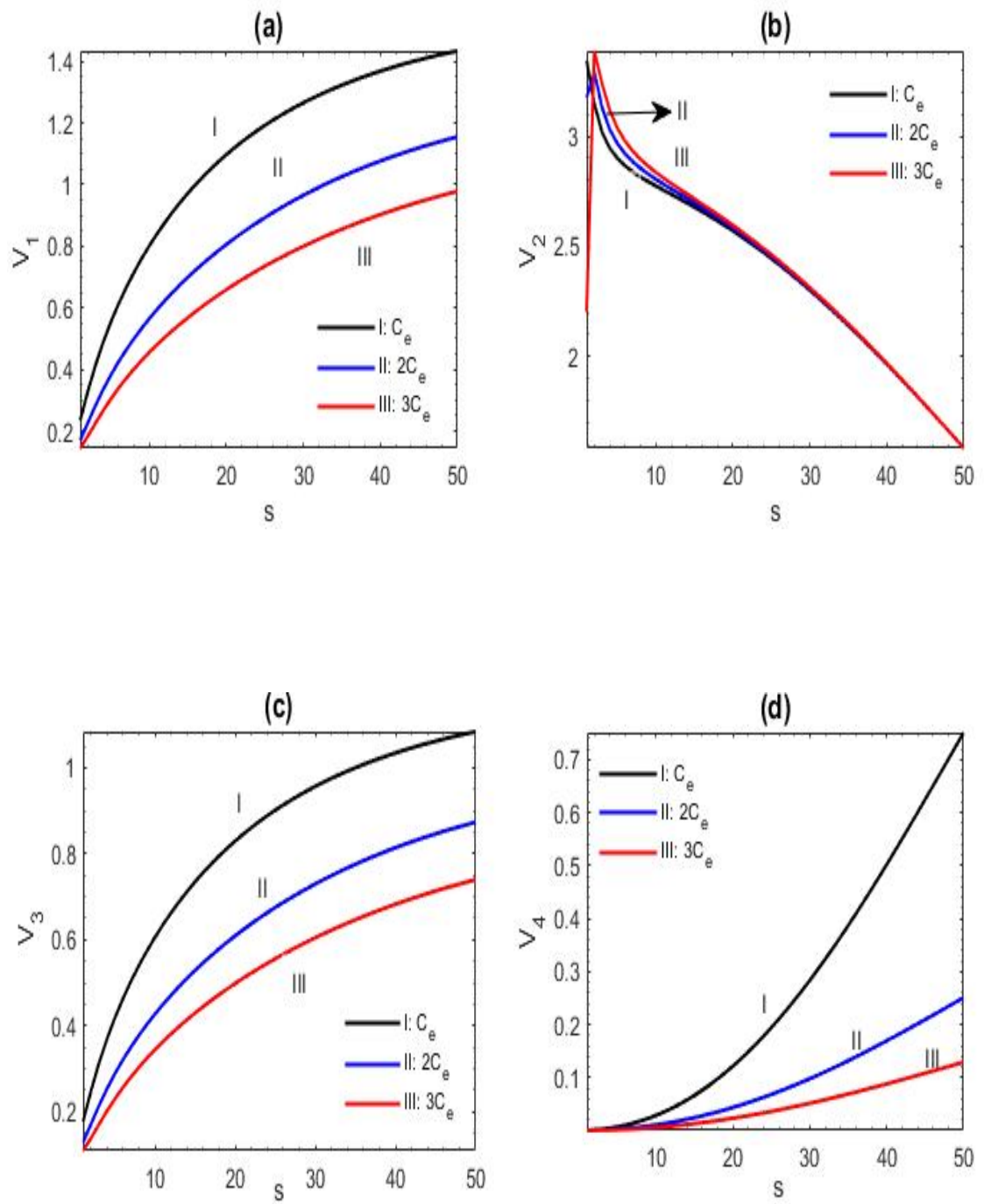
4.8 Numerical Discussion

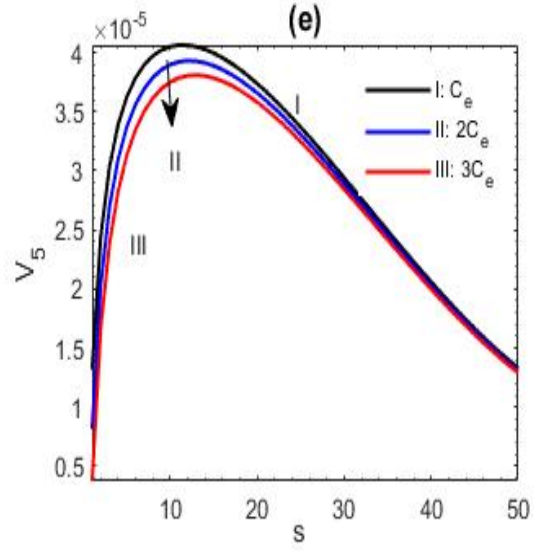
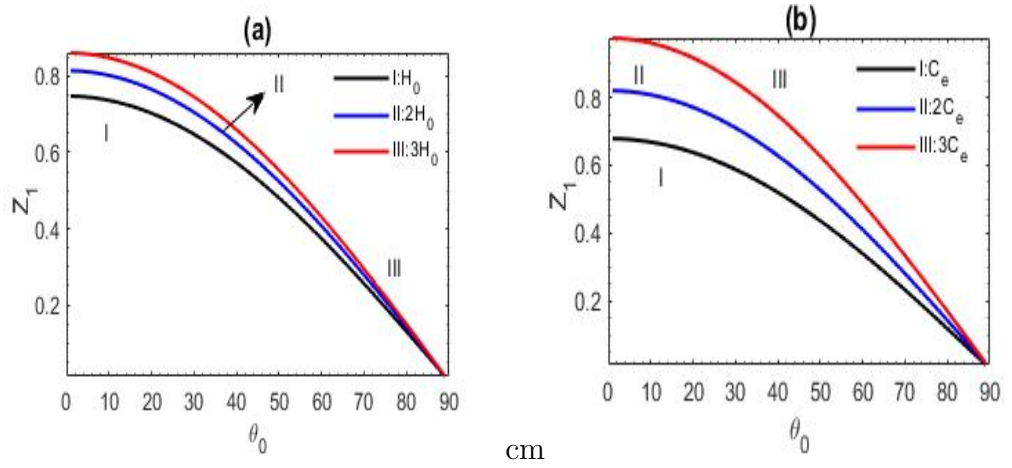
We have computed the phase velocity of the coupled five elastic waves and obtained the amplitude and energy ratios of the reflected waves for an incident longitudinal wave at a free surface of of generalized magneto micropolar thermoelastic material. For the numerical computation, the following relevant parameters are used (Ezzat and Awad, 2009)

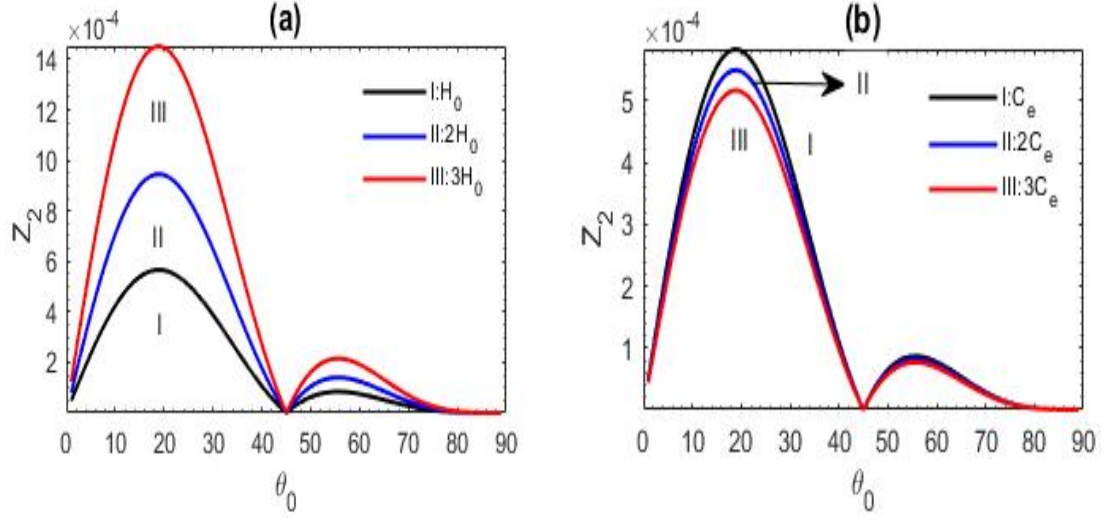
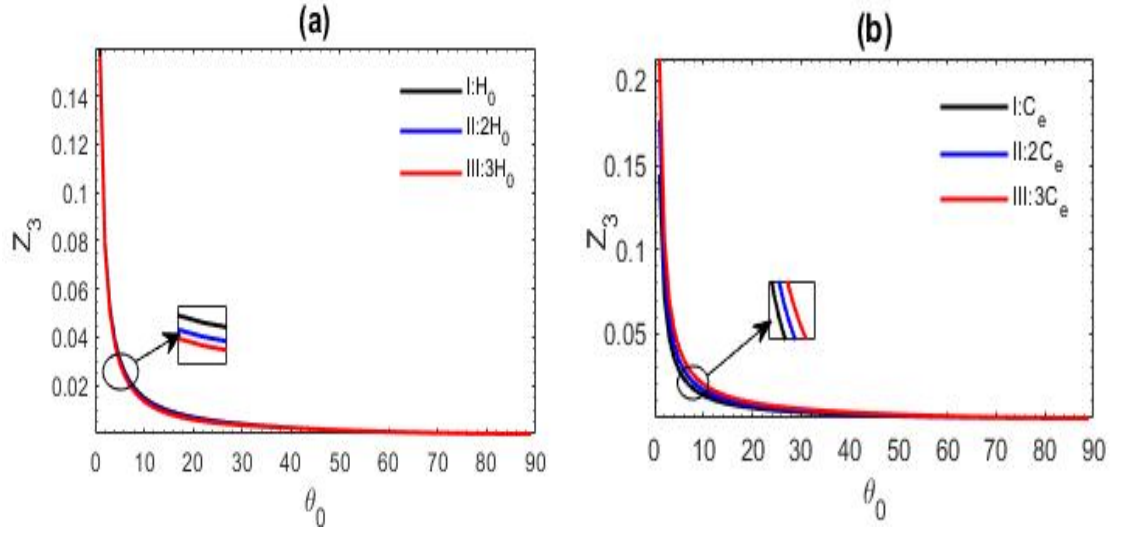
Parameters	Values	Units
λ	9.4×10^{11}	$dyne/cm^2$
α	4.5×10^{11}	$dyne/cm^2$
ρ	1.74	gm/cm^3
κ_0	0.6×10^{-2}	$Cal/cmsec^\circ C$
μ_0	$4\pi \times 10^{-2}$	$dyne \ sec^2/Col^2$
H_0	10^5	$Col/cm \ sec$
μ	4.5×10^{11}	$dyne/cm^2$
$\nu + \beta$	0.079×10^{-4}	$dyne/cm^2$
j	0.2×10^{-15}	cm^2
σ_0	9.36×10^5	$Col^2/Cal \ cm \ sec$
ϵ_0	$10^{-18}/36\pi$	$Col^2/dyne \ cm^2$
C_e	0.23	$Cal/gm^\circ C$

Table 4.1: Values of parameters

We have examined the effect of magnetic and thermal on the waves propagated in generalized magneto micropolar thermoelastic materials. The variation of non-dimensional phase velocity with the angular velocity, s is shown in Figure 4.2. Figures (4.3)-(4.7) represent the variation of amplitude ratios with θ_0 and those of energy ratios are represented by Figures (4.8)-(4.12). We have seen the effect of C_e on the phase velocity of coupled waves in Fig.(4.2).



Figure 4.2: Variation of phase velocities for different values of C_e Figure 4.3: Variation of Z_1 for diverse values of (a) H_0 and (b) C_e

Figure 4.4: Variation of Z_2 for diverse values of (a) H_0 and (b) C_e Figure 4.5: Variation of Z_3 for diverse values of (a) H_0 and (b) C_e

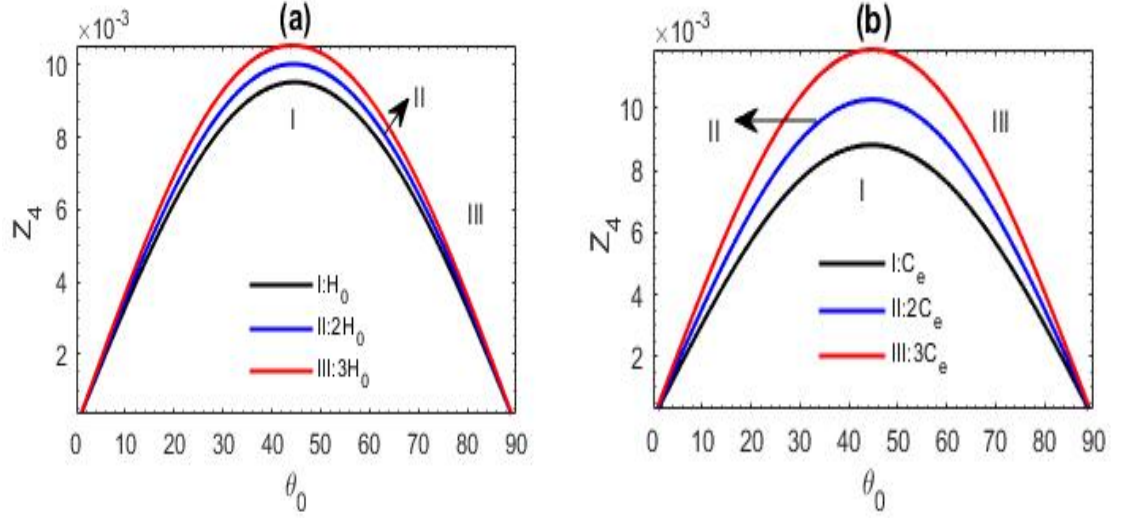


Figure 4.6: Variation of Z_4 for diverse values of (a) H_0 and (b) C_e

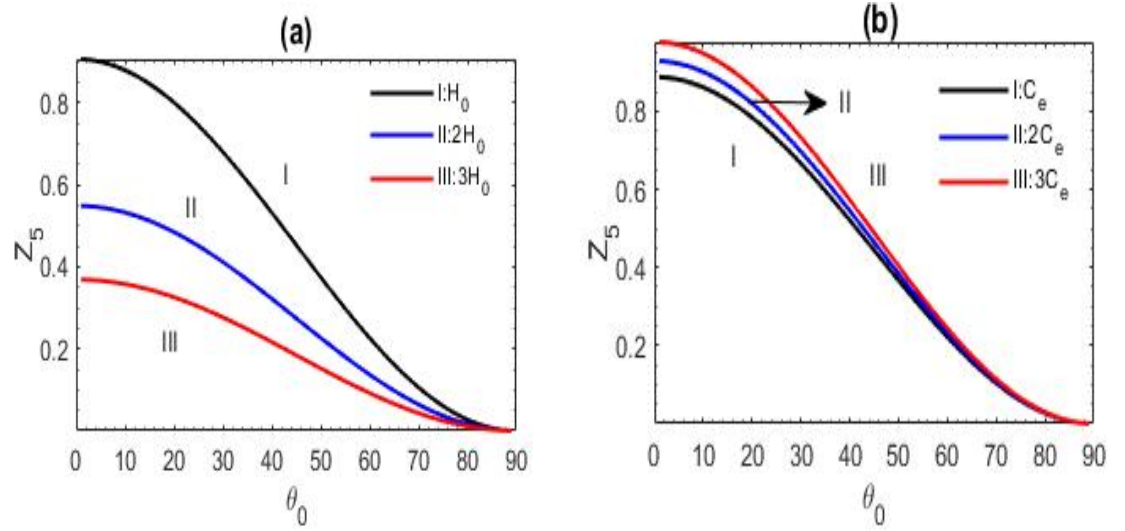
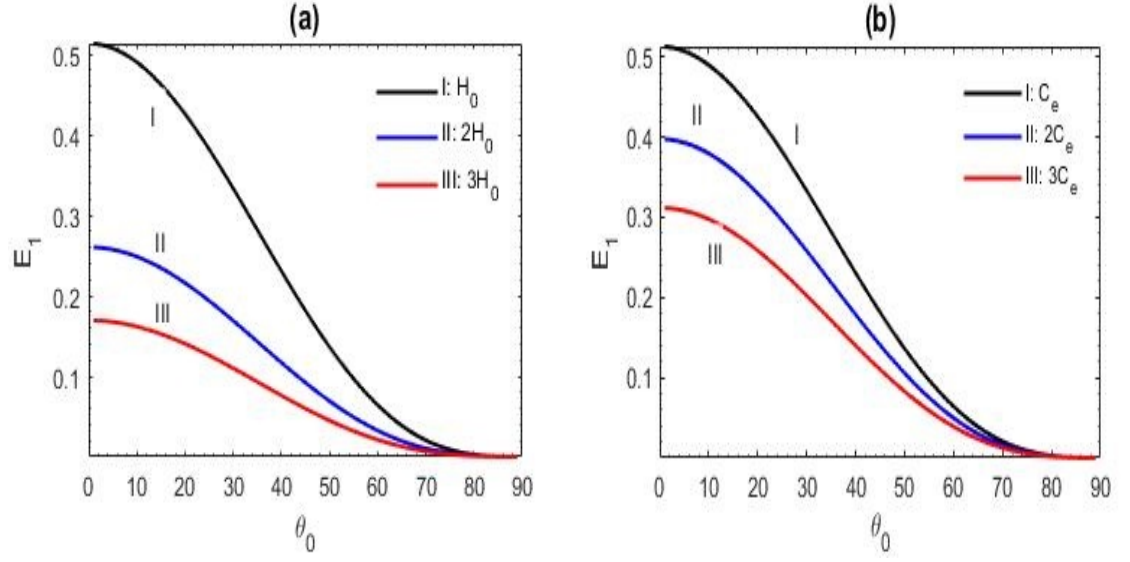
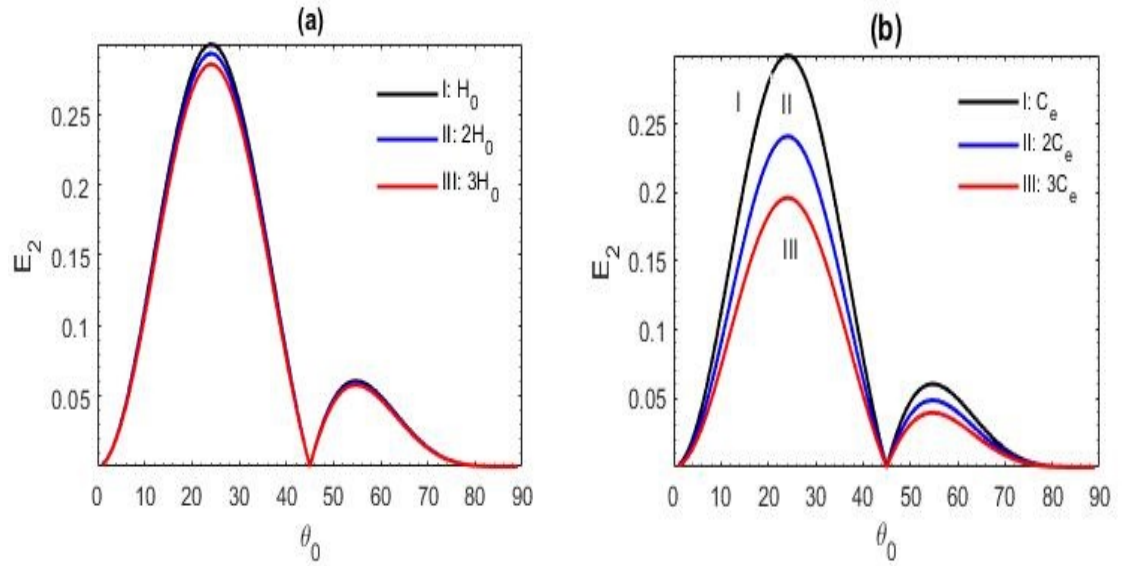
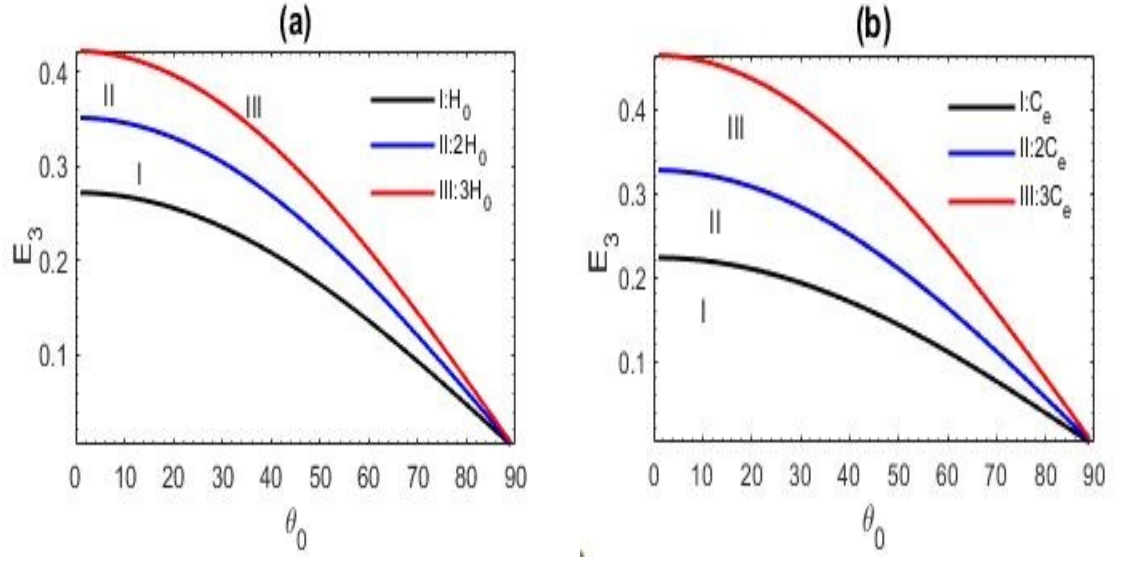
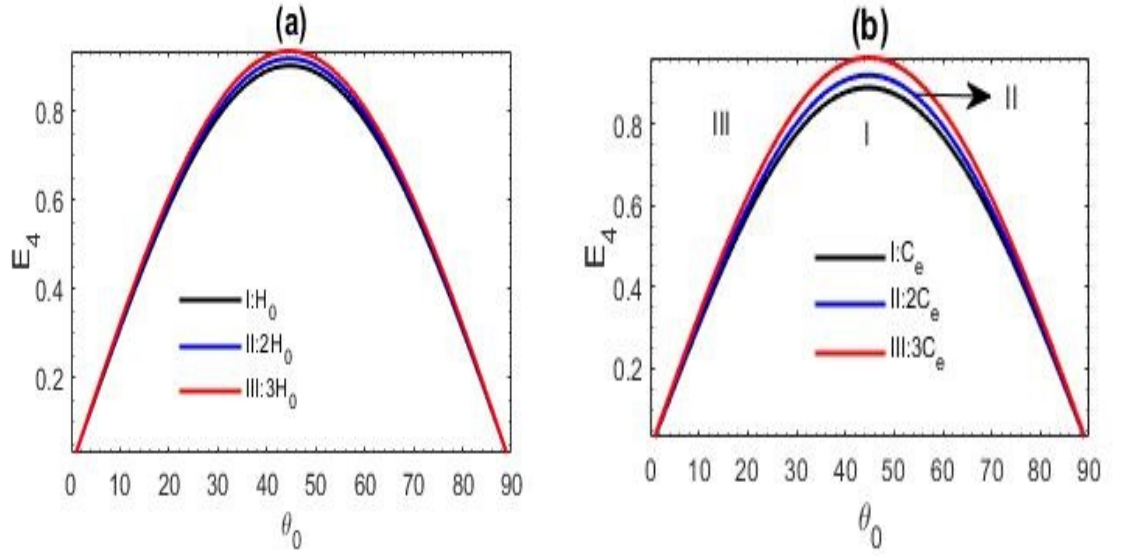
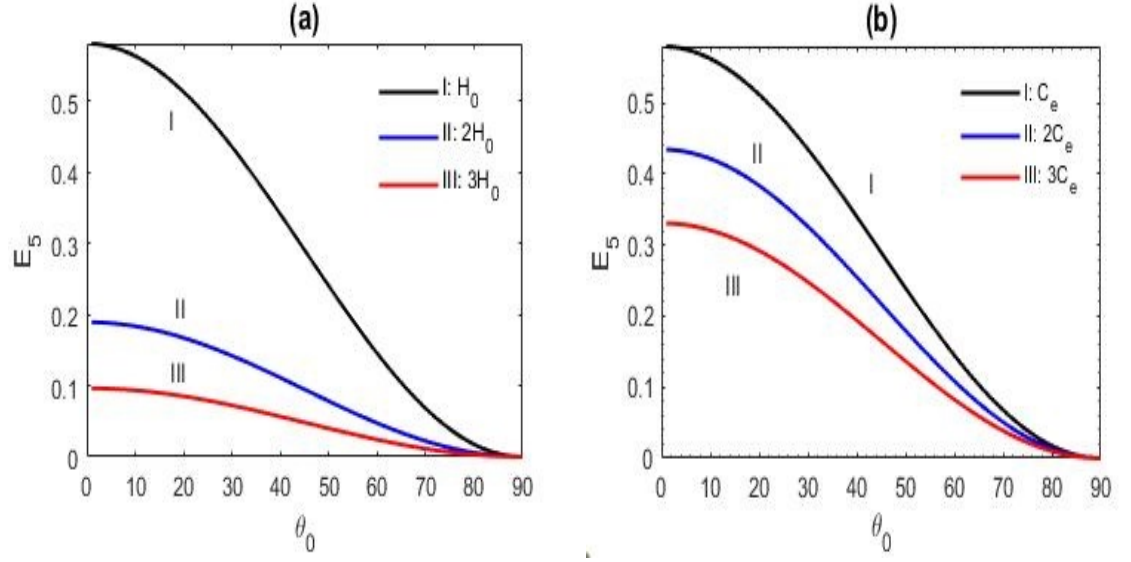
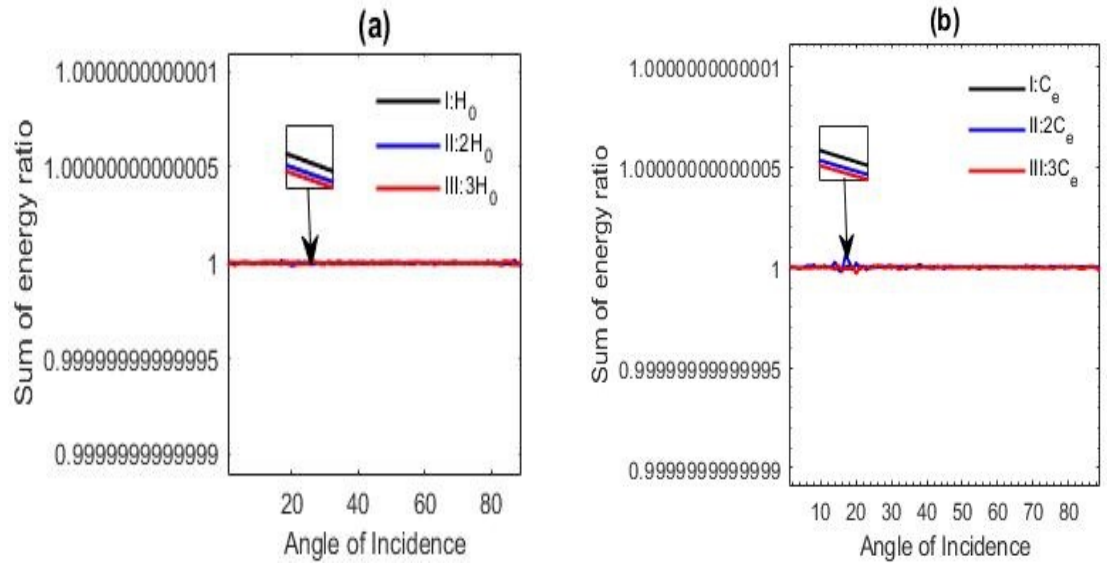


Figure 4.7: Variation of Z_5 for diverse values of (a) H_0 and (b) C_e

Figure 4.8: Variation of E_1 for diverse values of (a) H_0 and (b) C_e Figure 4.9: Variation of E_2 for diverse values of (a) H_0 and (b) C_e

Figure 4.10: Variation of E_3 for diverse values of (a) H_0 and (b) C_e Figure 4.11: Variation of E_4 for diverse values of (a) H_0 and (b) C_e

Figure 4.12: Variation of E_5 for diverse values of (a) H_0 and (b) C_e Figure 4.13: Variation of Sum of energy ratios for diverse values of (a) H_0 and (b) C_e

In Fig. 4.2(a), (c) and (d), the values of V_1 , V_3 and V_4 start from certain value and then increase with the increase of s . Fig.4.2(b) shows that V_2 starts from certain value which increases upto certain value and then decreases to the minimum value with the increase of s . In Figure 4.2(e), the values of V_5 increases initially to the maximum value at $s = 10$ and then decreases with the increase of s . It may be noted that the values of V_i , ($i = 1, 3, 4, 5$) decrease with the increase of C_e , while those of V_2 increase with the increase of C_e .

We have seen the effect of H_0 and C_e on the amplitude ratios and energy ratios of coupled waves in generalized magneto micropolar thermoelastic material. The variation of non-dimensional amplitude ratios and energy ratios with respect to angle of propagation is shown in Figures (4.3)-(4.12). In Figures 4.3, 4.5 and 4.7, the values of Z_1 , Z_3 and Z_5 start from certain value and decrease with the increase of θ_0 . The value of Z_2 in Figure 4.4 shows two parabolic regions in $0^\circ \leq \theta_0 \leq 45^\circ$ and $45^\circ \leq \theta_0 \leq 75^\circ$ with maximum values at $\theta_0 = 25^\circ$ and $\theta_0 = 55^\circ$ respectively. In Figure 4.6, the values of Z_4 increase initially to the maximum value at $\theta_0 = 55^\circ$ and then decrease with the increase of θ_0 . It has been found that the values of Z_1 , Z_2 , Z_5 and Z_3 , Z_4 decrease and increase respectively with the increase of H_0 and C_e . The effects of H_0 and C_e on Z_1 , Z_3 and Z_5 are maximum at normal angle of incidence and minimum at grazing angle of incidence. The effects of H_0 and C_e on Z_4 are maximum at $\theta_0 = 50^\circ$ and minimum at normal and grazing angle of incidence. In Figures (4.8)-(4.12), we have observed that the variation of E_i with θ_0 are similar with Z_i . We also confirmed that the sum of energy ratios in Figure 4.13 are close to unity.

4.9 Conclusion

We study the problem of reflection of waves in generalized magneto thermoelastic micropolar medium. We find out the expressions of amplitude and energy ratios of reflected waves by using appropriate boundary conditions. These ratios are functions of angle of incidence, micropolar, magnetic, thermal and elastic parameters. The present study shows the meaningful effects on the magnetic and thermal parameters on the propagating waves. The following points are concluded from the analysis:

- The values of phase velocity corresponding to V_i , ($i = 1, 3, 4, 5$) decrease with

the increase of C_e .

- The values of phase velocity, V_2 increase with the increase of C_e .
- The values of amplitude ratios, Z_1 , Z_2 and Z_5 decrease with the increase of H_0 and C_e .
- The values of amplitude ratios, Z_3 and Z_4 increase with the increase of H_0 and C_e . The behaviour of energy ratios, E_i of the refracted waves are similar with Z_i .
- The sum of energy ratios of the reflected waves are close to unity for different values of H_0 and C_e .

Chapter 5

P and SV-waves from a plane interface in micropolar generalized magneto-thermoelastic half spaces.

5.1 Introduction

The linear theory of elasticity is of great importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials like concrete, wood, and coal. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. Macroscopic characteristics of a material, such as temperature, stress, and strain, are used to characterize its behaviour. Ciarletta et al.(2009) studied the linear theory of micropolar thermoelasticity for isotropic and homogenous materials with voids using thermal relaxation time. Kumar and Gupta(2010) analyzed the problem of elastic wave propagation in transversely isotropic micropolar generalized thermoelastic half-space.

In this present work, we investigate a model of reflection/refraction of homogeneous plane waves at a plane interface of generalized magneto micropolar thermoelastic half-spaces using modified Ohm's law and generalized Fourier law. The phase velocities of these waves are obtained analytically and numerically. The amplitude and energy ratios corresponding to the reflected and refracted waves are obtained analytically and numerically with the use of suitable boundary conditions. The effects of magnetic and thermal on the amplitude and energy ratios are examined. The

conservation of energy through the reflected and refracted waves verify the model.

5.2 Wave propagation

Let us take y -axis of the Cartesian coordinate is lying vertically with pointing downward and x -axis as horizontally. Assume that two half-space of generalized magneto micropolar thermoelastic materials are separated by $y = 0$ as $\{M(x, y) : 0 \leq y < \infty\}$ and $\{M'(x, y) : -\infty < y \leq 0\}$.

The equations of motion in the half-space, M are given by Eqs. (4.13)-(4.17), while those of half-space, M' are represented by

$$(\nabla^2 - \frac{\partial^2}{\partial t^2} - \nu_1'^2 \epsilon_2' \frac{\partial}{\partial t})e' = \nabla^2 T' + \nu_1'^2 \epsilon_2' \frac{\partial h'}{\partial t}, \quad (5.1)$$

$$\left[(1 - \epsilon_3') \nabla^2 - \beta_1'^2 \left(\frac{\partial^2}{\partial t^2} + \nu_1' \epsilon_2' \frac{\partial}{\partial t} \right) \right] \phi' = (\beta_1'^2 \kappa_1' - \epsilon_{30}') \nabla^2 T' - 2 \nabla^2 w', \quad (5.2)$$

$$(\nabla^2 - V'^2 \frac{\partial^2}{\partial t^2} + \nu_1' \frac{\partial}{\partial t})h' = \frac{\partial e'}{\partial t}, \quad (5.3)$$

$$\left[(\nabla^2 - V'^2 \frac{\partial}{\partial t}) \frac{\iota s}{\nu_1' + \iota s V'^2} + \frac{\partial}{\partial t} \right] \phi' = \left[(\nabla^2 - V'^2 \frac{\partial}{\partial t}) \frac{\kappa_2'}{\nu_1' + \iota s V'^2} - \kappa_2' \right] \nabla^2 T', \quad (5.4)$$

$$\left[(1 + \pi_2') \nabla^2 - \left(\frac{\partial}{\partial t} + \tau_0' \frac{\partial^2}{\partial t^2} \right) \right] T' = \epsilon_1' \left(\frac{\partial}{\partial t} + \tau_0' \frac{\partial^2}{\partial t^2} \right) e' + \pi_{20}' \frac{\partial \phi'}{\partial t}, \quad (5.5)$$

where

$$\epsilon_3' = \frac{\beta_1'^2 \epsilon_2'}{\nu_1' + \iota s V'^2}, \quad \epsilon_{30}' = \frac{\beta_1'^2 \nu_1'^2 \epsilon_2' \kappa_2'}{\nu_1' + \iota s V'^2}, \quad \pi_2' = \frac{\kappa_2' \pi_1'}{\nu_1' + \iota s V'^2}, \quad \pi_{20}' = \frac{\iota s \pi_1'}{\nu_1' + \iota s V'^2}.$$

It may be noted that the parameters of M' like β_1' , ϵ_2' , ν' etc. are obtained by inserting $(')$ in the corresponding quantities of M . The expressions in the form of plane waves by expressing in the exponential form for the half-space, M are given by Eq. (4.23), while those of half-space, M' are given by

$$(e', \phi', u', v', w', T', h') = \sum_{i=0}^5 (1, \alpha_i', \Omega_i', \Delta_i', \eta_i', \gamma_i', \mu_i') A_i' \exp\{\iota \left(st - k_i'(xp_1'^{(i)} + yp_2'^{(i)}) \right)\}, \quad (5.6)$$

where A_i' are amplitude constants of the refracted waves and the coupling constants α_i' , Ω_i' , Δ_i' , η_i' , γ_i' and μ_i' are given by

$$\mu_i' = \frac{\alpha' \iota s}{g_6' - k_i'^2}, \quad \gamma_i' = \frac{(-k_i'^2 + g_6')(k_i'^2 - g_5') + \nu_1'^2 \epsilon_2' s^2}{(g_6' - k_i'^2) k_i'^2}, \quad \eta_i' = \frac{\alpha g_1' \alpha_i'}{g_4' - k_i'^2}, \quad \Delta_i' = \frac{p_2'^{(i)} - \alpha_i' p_1'^{(i)}}{\iota k_i' (p_2'^{(i)2} - p_1'^{(i)2})}$$

$$\alpha_i' = \frac{1}{g_7' \iota s} \{(-k_i'^2 + g_8' + k_i'^2 \kappa_2' g_4') \gamma_i' + g_9' \epsilon_1'\}, \quad \alpha_i' = \iota k_i' p_2'^{(i)} \Omega_i' - \iota k_i' p_1'^{(i)} \Delta_i'.$$

The Snell's law, for the problem, is given by

$$k_0 \sin \theta_0 = k_i \sin \theta_i = k'_i \sin \theta'_i, \quad (i = 1, 2, 3, 4, 5) \quad (5.7)$$

where θ_i and θ'_i are angles of reflected and refracted waves respectively.

5.3 Appropriate Boundary Conditions

The interface between the two continuum half-spaces is noted as the point where the boundary conditions are defined. There are continuity of components of displacement, microrotation, stress, coupled stress tensors and heat flux at $x = 0$. These conditions are given as

$$\begin{aligned} u = u', \quad v = v', \quad w = w', \quad \sigma_{xx} = \sigma'_{xx}, \quad \sigma_{xy} = \sigma'_{xy}, \quad (5.8) \\ \mu_{yz} = \mu'_{yz}, \quad h = h', \quad \mu_{xz} = \mu'_{xz}, \quad k \frac{\partial T}{\partial x} = k' \frac{\partial T'}{\partial x}, \quad k \frac{\partial T}{\partial y} = k' \frac{\partial T'}{\partial y}, \end{aligned}$$

where

$$\begin{aligned} \sigma_{xx} = \beta_1^2 \left(\frac{\partial u}{\partial x} - T \right) + \delta_6 \frac{\partial v}{\partial y}, \quad \sigma_{xy} = \frac{\partial v}{\partial x} + \delta_7 \frac{\partial u}{\partial y} - 2w, \quad \mu_{yz} = \frac{\partial w}{\partial y}, \quad \mu_{xz} = \frac{\partial w}{\partial x}, \\ \sigma'_{xx} = \beta_1'^2 \left(\frac{\partial u'}{\partial x} - T' \right) + \delta_6' \frac{\partial v'}{\partial y}, \quad \sigma'_{xy} = \frac{\partial v'}{\partial x} + \delta_7' \frac{\partial u'}{\partial y} - 2w', \quad \mu'_{yz} = \frac{\partial w'}{\partial y}, \quad \mu'_{xz} = \frac{\partial w'}{\partial x}. \end{aligned}$$

Using Eqs.(4.23) and (5.6) into (5.8), we get a matrix equation of the form

$$[a_i][Z] = [N], \quad (i : 1 \text{ to } 100) \quad (5.9)$$

where a_i is 10×10 matrix, $Z = [Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8, Z_9, Z_{10}]^t$ is the amplitude ratio with $Z_i = \frac{A_i}{A_0}$, $Z_{5+i} = \frac{A'_i}{A_0}$, $N = [N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}]^t$ and their expressions are given below:

$$\begin{aligned} a_i = \frac{\Omega_i}{\Omega_0}, \quad a_{5+i} = \frac{\Omega'_i}{\Omega_0}, \quad a_{10+i} = \frac{\Delta_i}{\Delta_0}, \quad a_{15+i} = \frac{\Delta'_i}{\Delta_0}, \quad N_i = N_{5+i} = -1, \\ a_{20+i} = \frac{\eta_i}{\eta_0}, \quad a_{25+i} = \frac{\eta'_i}{\eta_0}, \quad a_{30+i} = \frac{d_i}{d_0}, \quad a_{35+i} = \frac{d'_i}{d_0}, \\ a_{40+i} = \frac{f_i}{f_0}, \quad a_{45+i} = \frac{f'_i}{f_0}, \quad a_{50+i} = \frac{\eta_i k_i p_2^i}{\eta_0 k_0 p_2^0}, \quad a_{55+i} = \frac{\eta'_i k'_i p_2^i}{\eta_0 k_0 p_2^0}, \\ a_{60+i} = \frac{\mu_i}{\mu_0}, \quad a_{65+i} = \frac{\mu'_i}{\mu_0}, \quad a_{70+i} = \frac{\eta_i k_i}{\eta_0 k_0}, \quad a_{75+i} = \frac{(\nu' + \beta') k'_i \eta'_i}{(\nu + \beta) k_0 \eta_0} \end{aligned}$$

$$a_{80+i} = \frac{\gamma_i}{\gamma_0}, \quad a_{85+i} = \frac{k'\gamma'_i}{k\gamma_0}, \quad a_{90+i} = \frac{p_2^{(i)}k_ia_{80+i}}{p_2^{(0)}k_0}, \quad a_{95+i} = \frac{p_2'^{(i)}k'_ia_{85+i}}{p_2'^{(0)}k_0}, \quad (i = 1, 2, 3, 4, 5)$$

$$d_i = \Omega_i\beta_1^2k_0p_1^{(0)}\iota + \gamma_i\beta_1^2 + \delta_6\Delta_ik_i\iota p_{(2)}^{(i)}, \quad (i = 0 \text{ to } 5)$$

$$d'_i = \Omega'_i\beta_1'^2k_0p_1^{(0)}\iota + \gamma'_i\beta_1'^2 - \delta'_6\Delta'_ik'_i\iota p_{(2)}'^{(i)}, \quad (i = 1 \text{ to } 5)$$

$$f_i = \Delta_i\iota k_0p_1^{(0)} + \delta_7\Omega_i\iota k_1p_2^i + 2\eta_i, \quad f'_i = \Delta'_i\iota k_0p_1^{(0)} + \delta'_7\Omega'_i\iota k'_1p_2'^i + 2\eta'_i.$$

It may be noted that amplitude ratios depend on angle of propagation, elastic constants, micropolar, thermal and coupling parameters.

5.4 Energy Partition

Let us consider energy partition among reflected and refracted waves at the plane interface $y = 0$. The rate of energy transmission per unit area at $y = 0$ is given by (Tomar and Gogna, 1995; Tung, 2021; Khan et al., 2025)

$$\mathcal{P} = \langle \sigma_{xx}^i, \dot{u}^j \rangle + \langle \sigma_{xy}^i, \dot{v}^j \rangle + \langle \mu_{xz}^i, \dot{w}^j \rangle, \quad (i, j = 0, 1, 2, 3, 4, 5), \quad (5.10)$$

$$\mathcal{P}' = \langle \sigma_{xx}^i, \dot{u}^j \rangle + \langle \sigma_{xy}^i, \dot{v}^j \rangle + \langle \mu_{xz}^i, \dot{w}^j \rangle, \quad (i, j = 1, 2, 3, 4, 5) \quad (5.11)$$

where $i, j = 0$ represents for the incident wave, while $i, j = 1, 2, 3, 4, 5$ represent for the reflected and refracted waves. The energy distribution of the incident, reflected and refracted waves are given by

$$P_{inc} = A_0^2 s \{ p_1^{(0)} k_0 (\Omega_0 \Delta_0 (\beta_1^2 + 1) - \eta_0^2) + p_2^{(0)} k_0 (\Delta_0^2 \delta_6 + \delta_7 \Omega_0^2) - \iota (2\eta_0 \Omega_0 + \beta_1^2 \gamma_0 \Delta_0) \}, \quad (5.12)$$

$$P_r = A_i^2 s \{ p_1^{(0)} k_0 (\Omega_i \Delta_i (\beta_1^2 + 1) - \eta_i^2) + p_2^{(i)} k_i (\Delta_i^2 \delta_6 + \delta_7 \Omega_i^2) - \iota (2\eta_i \Omega_i + \beta_1^2 \gamma_i \Delta_i) \}, \quad (5.13)$$

$$P_{r'} = A_i'^2 s \{ p_1^{(0)} k_0 (\Omega'_i \Delta_i (\beta_1'^2 + 1) - \eta_i'^2) + p_2'^{(i)} k'_i (\Delta_i'^2 \delta'_6 + \delta'_7 \Omega_i'^2) - \iota (2\eta_i \Omega'_i + \beta_1'^2 \gamma'_i \Delta'_i) \}, \quad (5.14)$$

where

$$b_{r0} = \Delta_0^2 \delta_6 + \delta_7 \Omega_0^2, \quad d_{r0} = \Omega_0 \delta_0 (\beta_1^2 + 1) - \eta_0^2, \quad e_{r0} = -\iota (2\eta_0 \Omega_0 + \beta_1^2 \gamma_0 \delta_6).$$

$$b_{ri} = \Delta_i^2 \delta_6 + \delta_7 \Omega_i^2, \quad d_{ri} = \Omega_i \delta_i (\beta_1^2 + 1) - \eta_i^2, \quad e_{ri} = -\iota (2\eta_i \Omega_i + \beta_1^2 \gamma_i \delta_i),$$

$$b_{ri'} = \Delta_i'^2 \delta'_6 + \delta'_7 \Omega_i'^2, \quad d_{ri'} = \Omega'_i \delta'_i (\beta_1'^2 + 1) - \eta_i'^2, \quad e_{ri'} = -\iota (2\eta_i \Omega'_i + \beta_1'^2 \gamma'_i \delta'_i).$$

Thus, the energy ratios corresponding to reflected and refracted waves are respectively represented by E_i and E'_i ($i = 1, 2, 3, 4, 5$) are given by

$$E_i = \frac{p_2^{(i)} k_i b_{ri} + p_1^{(0)} k_0 d_{ri} + e_{ri}}{p_2^{(0)} k_0 b_{r0} + p_1^{(0)} k_0 d_{r0} + e_{r0}} Z_i^2, \quad (5.15)$$

$$E_{i'} = \frac{p_2^{(i')} k'_i b_{ri'} + p_1^{(0)} k_0 d_{ri'} + e_{ri'}}{p_2^{(0)} k_0 b_{r0} + p_1^{(0)} k_0 d_{r0} + e_{r0}} Z_i^2. \quad (5.16)$$

The sum of energy ratios at the boundary surface for the five each reflected and refracted waves is equal to unity. This shows the conservation law of energy for this specific problem.

5.5 Particular case

Case 1: If we neglect micropolar effect, the problem reduces to the reflection of waves from a plane free half-space of generalized magneto thermoelastic material. The results are perfectly matched with those of Singh and Boruah(2023) given in Chapter 2.

Case 2: If we neglect magnetic effect, the problem reduces to the problem of reflection of waves from the half-space of generalized thermoelastic material. The results are perfectly matched with the special case 1 of Singh and Boruah(2023) given in Chapter 2.

5.6 Numerical Discussion

We have computed the amplitude and energy ratios of the reflected and refracted waves through Matlab programming. The following relevant values of parameters for the generalized magneto thermoelastic solid are considered (Jatain et al., 2023; Kumar and Partab, 2023)

Parameters	Values	Parameters	Values
T_0	$298K$	T'_0	$800K$
λ	$3.64 \times 10^{10} Nm^{-2}$	λ'	$9.4 \times 10^{10} Nm^{-2}$
μ	$5.46 \times 10^{10} Nm^{-2}$	μ'	$4.0 \times 10^{10} Nm^{-2}$
ρ	$2330kgm^{-3}$	ρ'	$1740kgm^{-3}$
τ_0	$5 \times 10^{-5}s$	τ'_0	$0.02s$
α_t	4.14×10^{-4}	α'_t	$1.78 \times 10^{-5} K^{-1}$
C_e	$695m^2/Ks^2$	C'_e	$383.1m^2/Ks^2$
κ	$1 \times 10^{10} Nm^{-2}$	κ'	$386Nm^{-2}$
γ	$0.779 \times 10^{-9}N$	γ'	$0.7 \times 10^{-9}N$
j	$0.2 \times 10^{-15}m^2$	j'	$0.2 \times 10^{19}m^2$
μ_0	$4\pi \times 10^{-7}Nms^2/C^2$	μ'_0	$4\pi \times 10^{-7}Nms^2/C^2$
H_0	$10C$	H'_0	$1C$

Table 5.1: Values of the parameters

We have examined the effect of magnetic and thermal on the waves propagated in generalized magneto micropolar thermoelastic materials. Figures (5.1)-(5.5) represent the variation of amplitude ratios with θ_0 and those of energy ratios are represented by Figures (5.6)-(5.10). We have seen the effect of H_0 on the amplitude and energy ratios of coupled reflected and refracted waves in generalized magneto micropolar thermoelastic material. The variation of non-dimensional amplitude ratios and energy ratios with respect to reflected and refracted waves is shown in Figures (5.1)-(5.10). In Figures (5.1)-(5.5), the amplitude ratios of reflected wave Z_1 , Z_2 , Z_3 and Z_4 start from certain value and increase with the increase of H_0 . The value of Z_5 in Figure 5 for reflected wave decreases with the increase of H_0 . The amplitude ratios of the refracted wave, Z_i , ($i = 6, 7, 8, 9, 10$) increase with the increase of H_0 . Figures 5.6-5.10, we have observed that the variation of energy ratios, E_i of the reflected/refracted waves with θ_0 are similar with Z_i . We also confirmed that the sum of energy ratios in Figure 5.11 are close to unity.

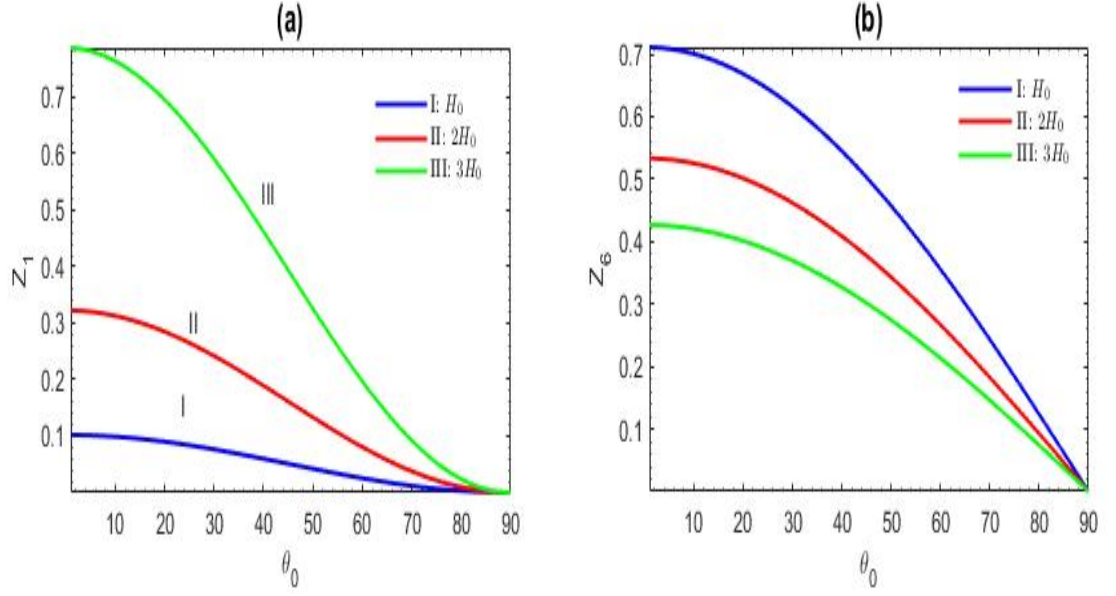


Figure 5.1: Variation of amplitude ratios for (a)reflected wave (Z_1) and (b)refracted wave (Z_6) for diverse value of H_0

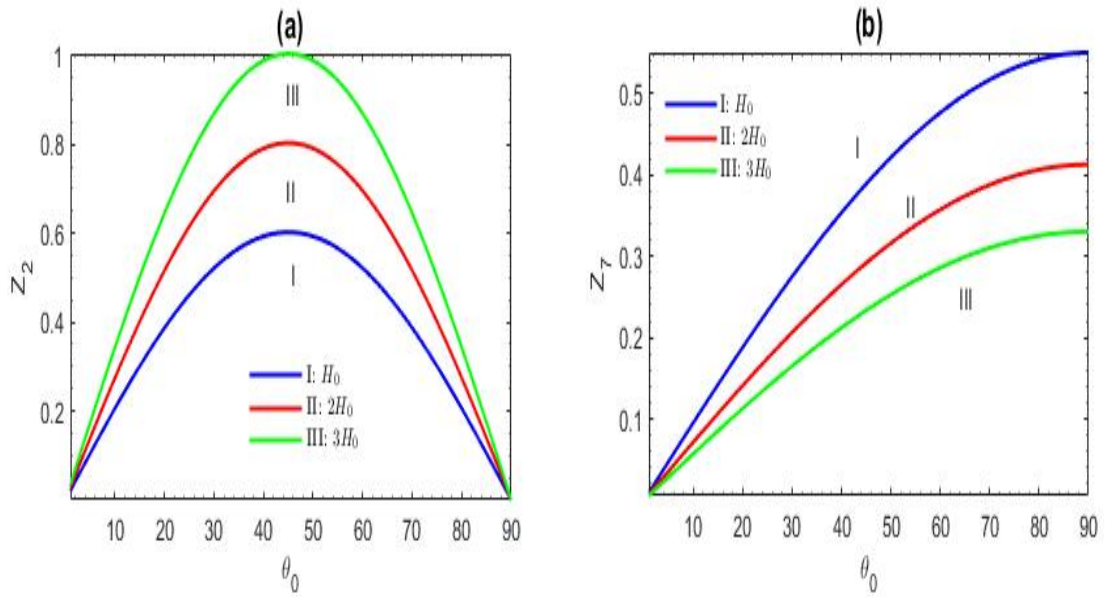


Figure 5.2: Variation of amplitude ratios for (a)reflected wave (Z_2) and (b)refracted wave (Z_7) for diverse value of H_0

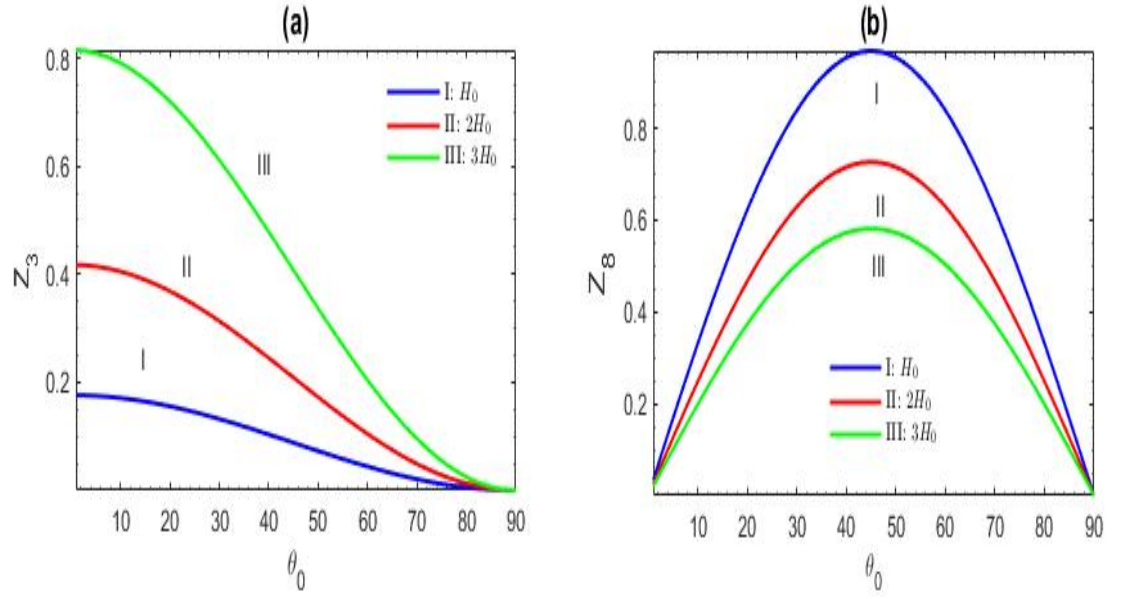


Figure 5.3: Variation of amplitude ratios for (a)reflected wave (Z_3) and (b)refracted wave (Z_8) for diverse value of H_0

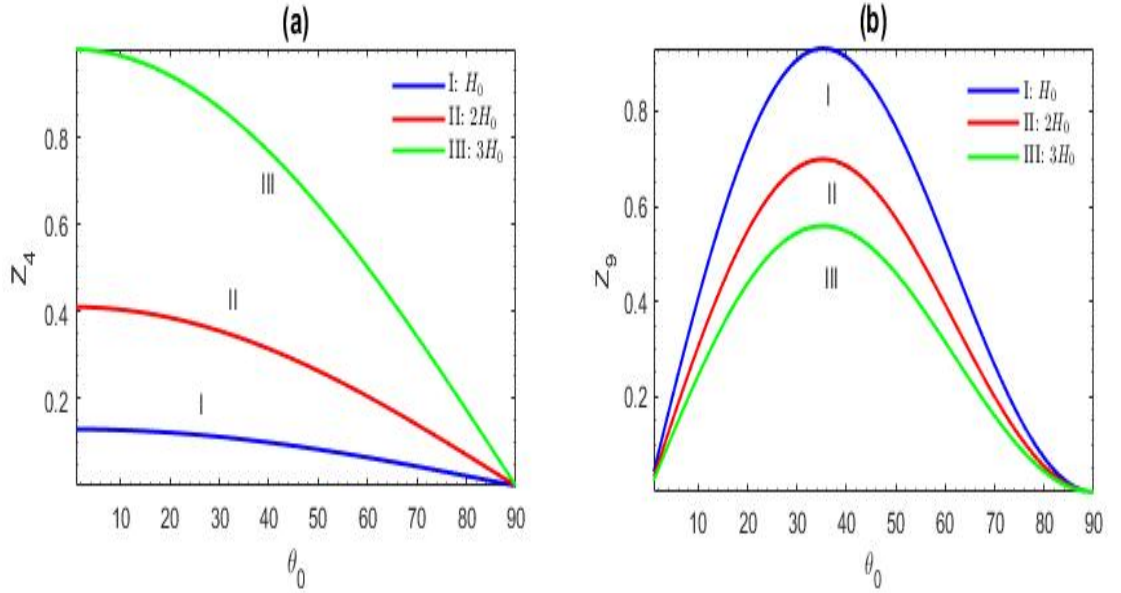


Figure 5.4: Variation of amplitude ratios for (a)reflected wave (Z_4) and (b)refracted wave (Z_9) for diverse value of H_0

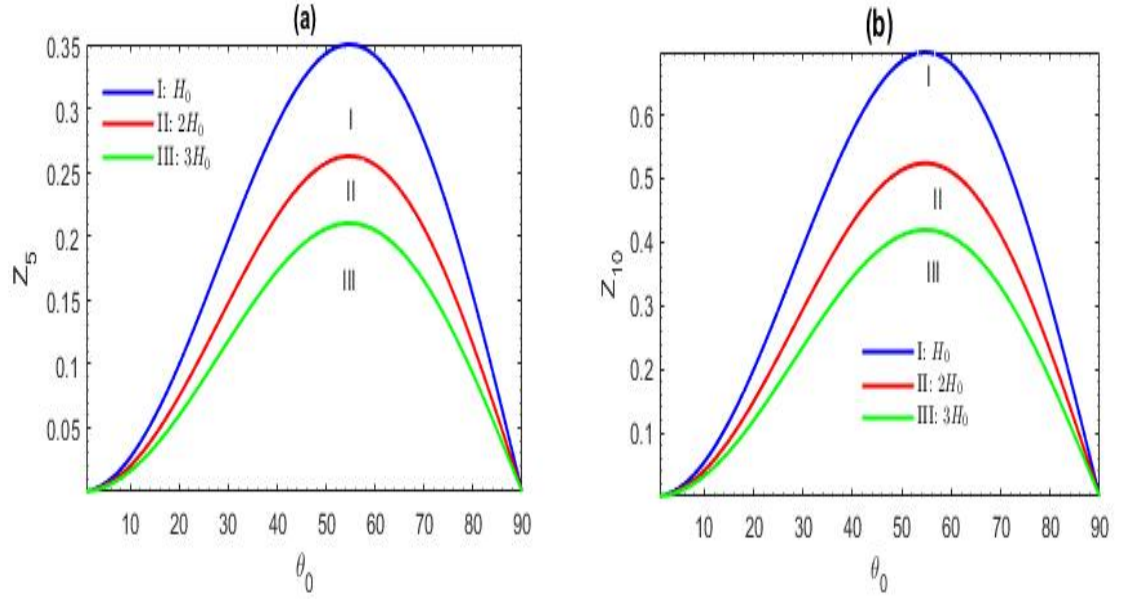


Figure 5.5: Variation of amplitude ratios for (a)reflected wave (Z_5) and (b)refracted wave (Z_{10}) for diverse value of H_0

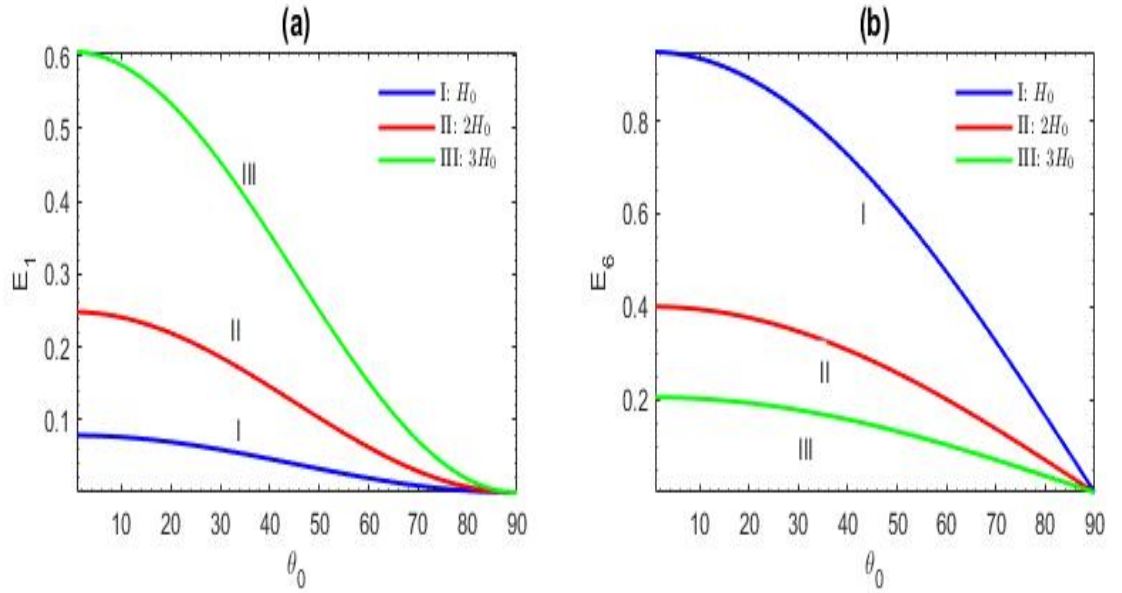


Figure 5.6: Variation of energy ratios for (a)reflected wave (E_1) and (b)refracted wave (E_6) for diverse value of H_0

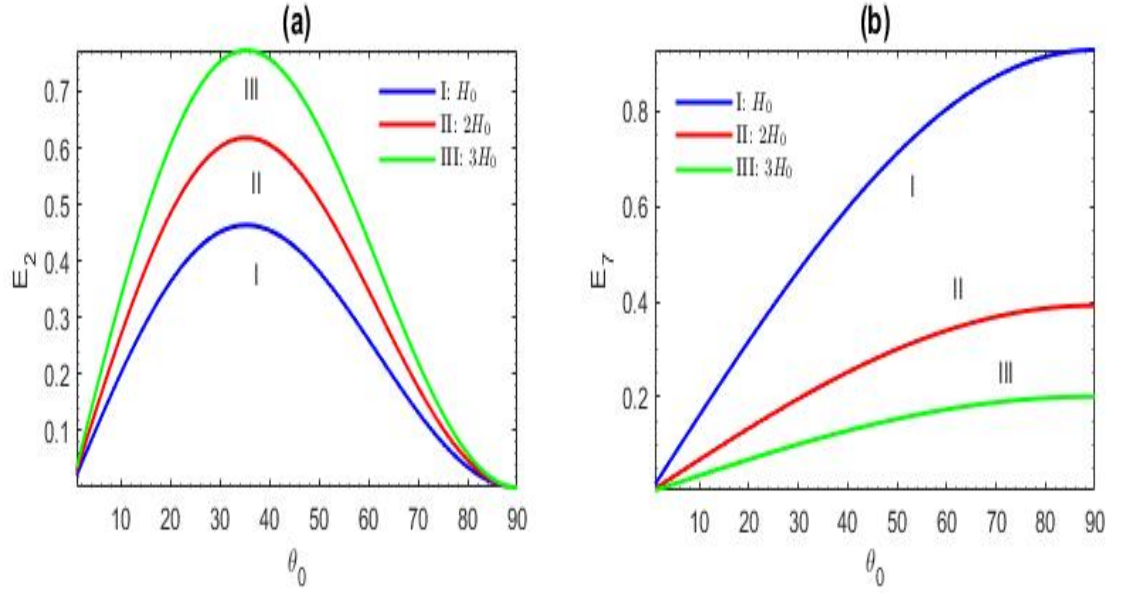


Figure 5.7: Variation of energy ratios for (a)reflected wave (E_2) and (b)refracted wave (E_7) for diverse value of H_0

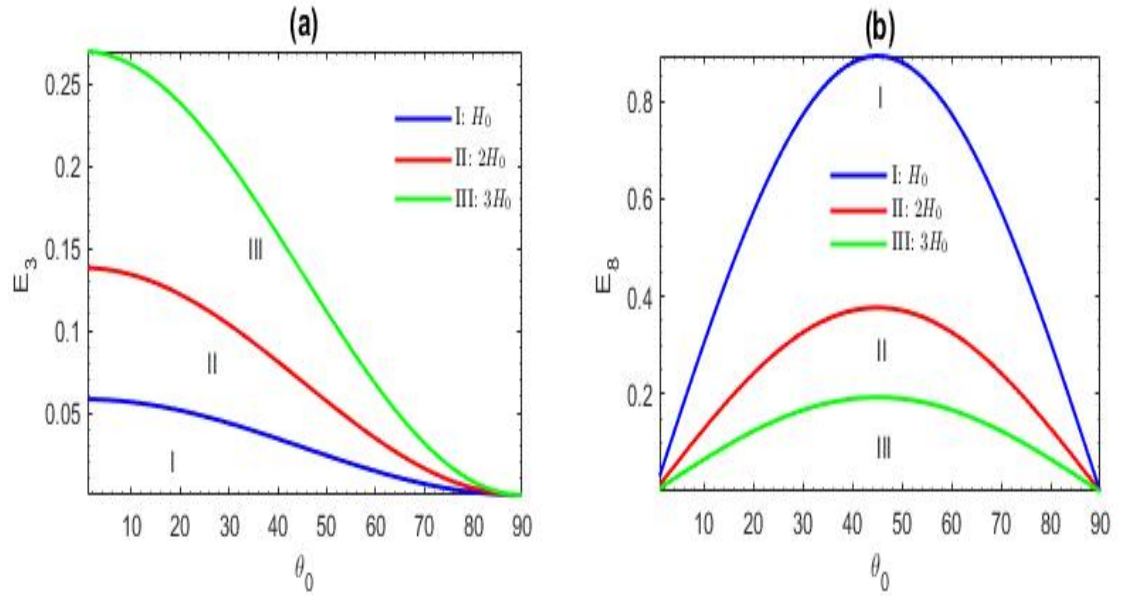


Figure 5.8: Variation of energy ratios for (a)reflected wave (E_3) and (b)refracted wave (E_8) for diverse value of H_0

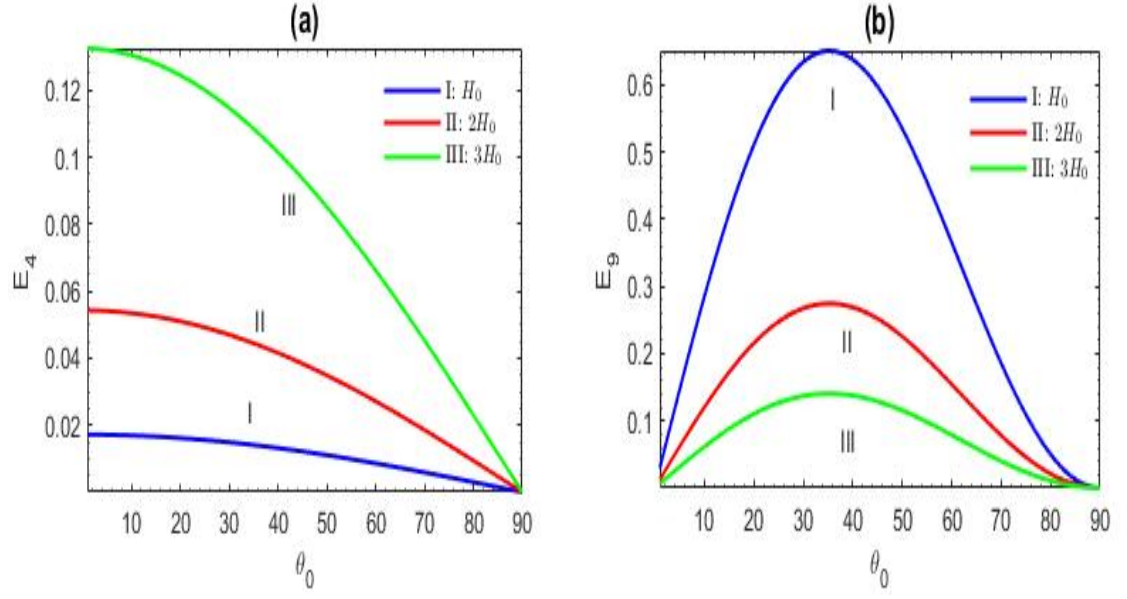


Figure 5.9: Variation of energy ratios for (a)reflected wave (E_4) and (b)refracted wave (E_9) for diverse value of H_0

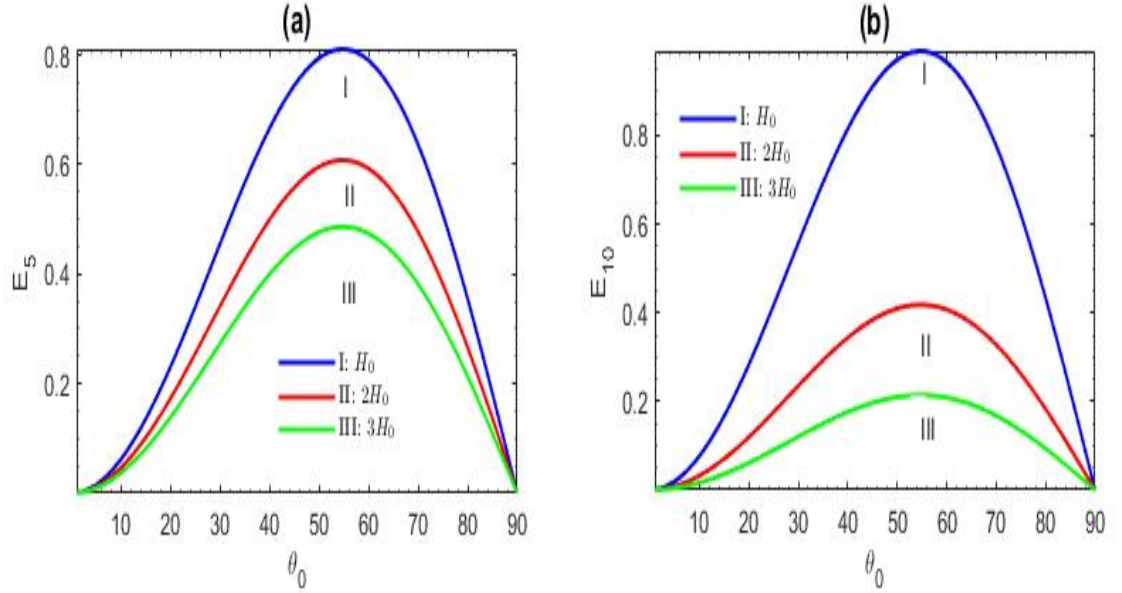
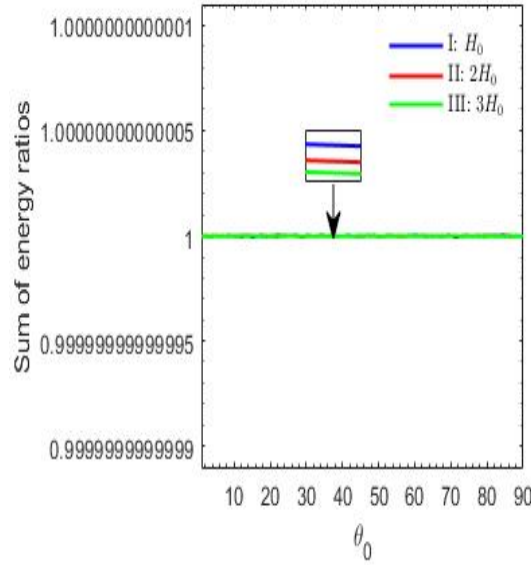


Figure 5.10: Variation of energy ratios for (a)reflected wave (E_5) and (b)refracted wave (E_{10}) for diverse value of H_0

Figure 5.11: Sum of energy ratios for diverse value of H_0

5.7 Conclusions

We study the problem of reflection and refraction of elastic waves in generalized magneto thermoelastic micropolar half-spaces. We find out the expressions of amplitude and energy ratios of reflected and refracted waves by using appropriate boundary conditions. These ratios are functions of angle of incidence, micropolar, magnetic, thermal and elastic parameters. The present study shows the meaningful effects on the magnetic parameters on the propagating waves. In numerical analysis, we have observed less effect of thermal parameter C_e on the amplitude and energy ratios of the reflected and refracted waves. The following points are concluded from the analysis:

- The values of amplitude ratios for reflected waves, Z_1 , Z_2 , Z_3 and Z_4 increase with the increase of H_0 and Z_5 decreases with the increase of H_0 .
- The values of amplitude ratios for refracted waves, Z_i ($i = 6, 7, 8, 9, 10$) decrease with the increase of H_0 .
- The effects of H_0 on E_i are minimum near the grazing angle of incidence. The behaviour of energy ratios, E_i of the reflected/refracted waves are similar with Z_i .

- The sum of the all energy ratios are close to unity for different values of H_0 .

Chapter 6

Summary and Conclusions

The present thesis comprises six chapters and focuses on the propagation of elastic waves in generalized magneto thermoelastic material and generalized magneto micropolar thermoelastic materials. We examine the behaviour of body waves as they travel through different thermoelastic media and derive the equations for amplitude ratios and energy ratios by applying suitable boundary conditions. The phenomena of reflection and refraction of elastic waves are also discussed in details. The first chapter provides a comprehensive introduction to the thesis. It outlines the fundamental concepts of base vectors, continuum concept, elastic waves, reflection and refraction at plane interface, thermoelasticity, Maxwell's Equations, Modified Ohm's law, constitutive relation and review of relevant literature.

In Chapter 2, the problem of wave reflection in generalized magneto-thermoelastic materials is studied. By applying boundary conditions, we have derived amplitude and energy ratios for reflected waves and analyzed them numerically. The amplitude and energy ratios are functions of angle of incidence, magnetic, thermal and elastic parameters. The results show that magnetic effects and thermal effects has influence in amplitude ratios and energy ratios. Longitudinal and thermal wave velocities increase with magnetic parameters and thermal parameters. The phase velocity of longitudinal wave increases with the increase of H_0 and C_e . The phase velocity corresponding to thermal wave increases with the increase of ω . The variations of Z_1 and Z_2 are similar in nature. The effect of H_0 on Z_i is very small. The effects of H_0 and C_e on E_i are minimum near the grazing angle of incidence. The sum of energy ratios are close to unity which verifies the conservation law of energy.

Chapter 3 deals with the problem of 3D wave propagation in a generalized magneto-thermoelastic medium using the eigenvalue technique to derive expressions for dis-

placement components and stress tensors, which are then computed numerically for a specific model. The effects of the magnetic field strength H_0 and the relaxation time, τ_0 on the displacement, stress tensors, and temperature field were investigated graphically. It was observed that the influence of τ_0 on displacement and stress is minimum near the origin, and similarly, the impact of H_0 on displacement components and the temperature field is also negligible. The stress components σ_{xz} and σ_{yz} were increase with increasing value of z , as well as with the increasing values of H_0 and τ_0 . In contrast the displacement components u , v and w and the normal stress σ_{zz} decreases with the value of H_0 and τ_0 increase. Additionally, similar patterns have been observed in u , v and σ_{zz} , as well as v and the temperature field, T .

In Chapter 4, we investigate the problem of wave reflection in a generalized magneto-thermoelastic micropolar thermoelastic material and derive expressions for the amplitude and energy ratios of reflected waves by applying the boundary conditions. These ratios depend on the angle of incidence as well as micropolar, magnetic, thermal, and elastic parameters. The study reveals significant effects of magnetic and thermal parameters on the behaviour of propagating waves. It is observed that the phase velocities corresponding to V_i , ($i = 1, 3, 4, 5$) decrease with an increase in the elastic constant C_e , while the phase velocity V_2 increases with increasing C_e . The amplitude ratios Z_1 , Z_2 and Z_5 show a decreasing pattern with increasing values of magnetic field, H_0 and thermal parameter, C_e and the amplitude ratios, Z_3 and Z_4 increase with the same parameters. Additionally, the sum of the energy ratios of the reflected waves remains close to unity for various values of H_0 and C_e , indicate that conservation of energy in the wave reflection phenomenon.

Chapter 5 discusses the reflection and refraction of elastic waves in generalized magneto-thermoelastic micropolar half-spaces, deriving expressions for amplitude and energy ratios based on boundary conditions. These ratios depend on the angle of incidence and various material parameters. Amplitude ratios, Z_1 , Z_2 , Z_3 , Z_4 corresponding to reflected waves increase with H_0 , while Z_5 and all amplitude ratios, Z_6 to Z_{10} corresponding to refracted waves decrease. The influence of H_0 on energy ratios E_i is minimum near grazing incidence, and the energy ratios follow trends similar to the amplitude ratios. The total energy ratio remains close to unity for all H_0 values.

The thesis is summarized with Summary and Conclusions in the sixth chapter.

Finally, a list of references is given at the end.

LIST OF PUBLICATIONS

1. Singh, S.S. and Boruah S.(2023). "Effect of Magnetic and Thermal Parameters on the Reflected Waves in Generalized Magneto Thermoelastic Materials", *Materials Plus*, **2**, 33-45.
2. Boruah S. and Singh, S.S.(2024). "3D Analysis of Wave Propagation in Generalized Magneto Thermoelastic Medium", *Science and Technology Journal*, **12(3)**, 38-44.
3. Debnath, S., Singh, S.S. and Boruah S.(2024). "Rayleigh waves in thermally affected skin tissues based on age, gender and morphology", *Computers in Biology and Medicine*, **179**, 108916.
4. Boruah S. and Singh, S.S.(2025). "Effect of magnetic and thermal parameters on the propagation of waves in generalized magneto micropolar thermoelastic medium", communicated with *Continuum Mechanics and Thermodynamics*.
5. Boruah S. and Singh, S.S.(2025). "Effect of magnetic and thermal parameters on reflection/refraction of elastic waves in generalized magneto micropolar thermoelastic medium", communicated with journal.

CONFERENCES/ SEMINARS/ WORKSHOPS

1. Attended *One week training program on Mathematical Modelling and Computing* organized by Mizoram University, Aizawl-796 004, Mizoram on 26th April to 2nd May, 2022.
2. Attended *International Faculty Development Programme on Mathematical Modelling of Biosystems with special focus on Epidemiology* organized by Mizoram University, Aizawl-796 004, Mizoram on 22th-27th August, 2022.
3. Attended *Teacher's Enricment Workshop on Groups, Rings and Number Theory* organized by Mizoram University, Aizawl-796 004, Mizoram on 12th-17th December, 2022.
4. Presented a paper "Effect of magnetic and thermal parameters on the reflected waves in generalized magneto thermoelastic materials" in the *National Conference on Advances in Mathematical Sciences* organized by Gauhati University, Guwahati-781 014, Assam on 22nd-23rd December, 2022.
5. Presented a paper "A two dimensional problem for a half space in generalized thermoelastic material" in the *National Conference on Recent Developments in Mathematics and Computer Science* organized by Government Champhai College, Champhai-796 321, Mizoram on 13th-14th March, 2023.
6. Presented a paper "Three Dimensional Elastic Waves in Generalized Thermoelastic Medium" in the *International Conference on Recent Advances in Mathematical, Physical, and Chemical Sciences (ICRAMPC-2024)* organized by Mizoram University, Aizawl 796 004, India on 21st-23rd February, 2024.
7. Attended *One-Day Awarness Programme on Orchid and Inflibnet Services for Scholarly Communities* jointly organized by the INFLIBNET Centre, Gandhinagar, Gujarat in collaboration with Mizoram University, Aizawl-796 004, Mizoram on 22nd May, 2024.

8. Mentor in *Initiation into Mathematics(Intimath) 2024* held at Mizoram University, Aizawl 796 004, India organized by MTTS Trust during 8th-13th July, 2024.
9. Presented a paper "Effect of magnetic parameter on the reflected waves in generalized magneto micropolar thermoelastic materials" in the 39th *Annual Conference of Ramanujan Mathematical Society (RMS-2024)* organized by CHRIST (Deemed to be University), Bengaluru, India on 27th-29th December, 2024.
10. Presented a paper "Effect of magnetic and themal parameters on reflection/refraction of elastic waves in generalized magneto micropolar thermoelastic medium" in the *International Conference on Recent Mathematical Sciences and Engineering (ICMSE-2025)* organized by Mizoram University, Aizawl 796 004, India on 9th-11th June, 2025.

Bibliography

- Abbas, I.A.(2015). "Eigenvalue approach to fractional order generalized magneto-thermoelastic medium subjected to moving heat source", *J. Magn. Magn. Mater.* **377**, 452-459.
- Abbas, I.A., Abd-Alla, A.E. and Othman, M.I.(2011). "Generalized magneto-thermoelasticity in a fiber-reinforced anisotropic half-space", *Int. J. Thermophys.* **32**, 1071-1085.
- Abbas, I.A. and Youssef, H.M.(2009). "Finite element analysis of two-temperature generalized magneto-thermoelasticity", *Arch. Appl. Mech.* **79**, 917-925.
- Abo-Dahab, S.M.(2014). "Green Lindsay model on propagation of surface waves in magneto-thermoelastic materials with voids and initial stress", *J. Comput. Theor. Nanosci.* **11(3)**, 763-771.
- Abo-Dahab, S.M.(2015). "Propagation of Stoneley waves in magneto-thermoelastic materials with voids and two relaxation times", *J. Vib. Control.* **21(6)**, 1144-1153.
- Abo-Dahab, S.M.(2018). "Reflection of generalized magneto-thermoelastic waves with two temperatures under influence of thermal shock and initial stress", *J. Heat Transfer.* **140(10)**, 102005(1-8).
- Abo-Dahab, S.M., Jahangir, A. and Abd-Alla, A.N.(2020). "Reflection of plane waves in thermoelastic microstructured materials under the influence of gravitation", *Contin. Mech. Thermodyn.* **32**, 803-815.

- Abo-Dahab, S.M. and Lotfy, K.(2015). "Generalized magneto-thermoelasticity with fractional derivative heat transfer for a rotation of a fibre-reinforced thermoelastic", *J. Comput. Theor. Nanosci.* **12(8)**, 1869-1881.
- Abouelregal, A.E.(2011). "Rayleigh waves in a thermoelastic solid half space using dual-phase-lag model", *Int. J. Eng. Sci.* **49(8)**, 781-791.
- Abouelregal, A.E., Moaaz, O. and Khalil, K.M., Abouhawwash, M. and Nasr, E.M.(2023). "Micropolar thermoelastic plane waves in microscopic materials caused by Hall-current effects in a two-temperature heat conduction model with higher-order time derivatives", *Arch. Appl. Mech.* **93**, 1901–1924.
- Abd-Alla, A.M.(1999). "Propagation of Rayleigh waves in an elastic half-space of orthotropic material", *Appl. Math. Comput.* **99(1)**, 61-69.
- Abd-Alla, A.M., Othman, M.I. and Abo-Dahab, S.M.(2016). "Reflection of plane waves from electro-magneto-thermoelastic half-space with a dual-phase-lag model", *CMC.* **51(2)**, 63-79.
- Abd-Alla, A.N., Al-sheikh, F.A. and Al-Hossain A.Y.(2012). "The reflection phenomena of quasi-vertical transverse waves in piezoelectric medium under initial stresses", *Meccanica.* **47**, 731-744.
- Abd-Alla, A.N. and Asker N.(2016). "Numerical simulations for the phase velocities and the electromechanical coupling factor of the Bleustein–Gulyaev waves in some piezoelectric smart materials", *Math. Mech. Solids.* **21(5)**, 539-551.
- Abd-Alla, A.N., Giorgio, I., Galantucci, L., Hamdan, A.M. and Vescovo, D.D.(2016). "Wave reflection at a free interface in an anisotropic pyroelectric medium with nonclassical thermoelasticity", *Contin. Mech. Thermodyn.* **28**, 67-84.

- Abd-Alla, A.N., Hamdan, A.M., Giorgio, I. and Del Vescovo, D.(2014). "The mathematical model of reflection and refraction of longitudinal waves in thermopiezoelectric materials", *Arch. Appl. Mech.* **84**, 1229-1248.
- Abd-Alla, A.N., Othman, M.I.A. and Hasbullah, N.F.(2019). "Shear horizontal waves in composite materials: Behavior under rotation and initial stress", *Math. Mech. Solids.* **24(1)**, 85-97.
- Abd-Alla, A.N., Yahia, A.A. and Abo-Dahab, S.M.(2003). "On the reflection of the generalized magneto-thermo-viscoelastic plane waves", *Chaos Soliton frac.* **16(2)**, 211-231.
- Achenbach, J.D.(1976). *Wave propagation in elastic solids*, North-Holland Publishing Company, New York.
- Agarwal, V.(1978). "On surface waves in generalized thermoelasticity", *J. Elast.* **8(2)**, 171-177.
- Ailawalia, P., Kumar, S. and Singh, D.(2016). "Internal heat source in thermoelastic micro elongated solid under green Lindsay theory", *J. Theor. Appl. Mech.* **46(2)**, 65-82.
- Ala, S., Gurijala, R. and Perati, M.R.(2020). "Shear wave propagation in magneto poroelastic medium sandwiched between self-reinforced poroelastic medium and poroelastic half space", *Eng. Comp.* **37(9)**, 3345-3359.
- Alharbi, A.R., Almatrafi, M.B. and Lotfy, K.(2020). "Constructions of solitary travelling wave solutions for Ito integro-differential equation arising in plasma physics", *Results Phys.* **19**, 103533.
- Altenbach, H. and Öchsner, A.(2020). *Encyclopedia of continuum mechanics*, Springer-Verlag, Germany.

- Anwar, M.N. and Sherief, H.H.(1988) "State space approach to generalized thermoelasticity", *J. Therm. Stresses*. **11(4)**, 353-365.
- Barber, J.R.(2010). *Elasticity(Solid Mechanics and Its Applications)*, Kluwer Academic Publishers, Dordrecht, 2nd Edition.
- Ben-Menahem A. and Singh, S.J.(1981). *Seismic Waves and Sources*, Springer, New Work.
- Biot, M.A.(1956). "Thermoelasticity and irreversible thermodynamics", *J. Appl. Phys.* **27(3)**, 240-253.
- Biot, M.A.(1965). *Mechanics of Incremental Deformations*, Wiley, New York.
- Boley, B.A. and Weiner J.H.(1960). *Theory of Thermal Stresses*, John Wiley and Sons, New Work.
- Borcherdt, R.D.(1982). "Reflection-refraction of general P and type-I S waves in elastic and anelastic solids", *Geophys. J. Int.* **70**, 621-638.
- Boschi, E. and Ieşan, D.(1973). "A generalized theory of linear micropolar thermoelasticity", *Meccanica*. **8**, 154-157.
- Brekhovskikh, L.M.(1960). *Waves in layered media*, New York, Academic Press.
- Bullen, K.E.(1963). *An introduction to the theory of seismology*, Cambridge, University Press.
- Cagniard, L.(1962). *Reflection and refraction of progressive waves*, translated and revised by E. A. Flinn and C. H. Dix. New York, McGraw-Hill Book Company.
- Chakraborty, N. and Singh M.C.(2011) "Reflection and refraction of a plane thermoelastic wave at a solid-solid interface under perfect boundary condition, in presence of normal initial stress", *Appl. Math. Model.* **35**, 5286–5301.

- Chandavath, B., Modem, R. and Perati, M.R.(2023). "Study of Curvature effect on stoneley wave propagation in dissipative poroelastic cylindrical solids", *J. Theor. Appl. Mech.* **53**, 34-48.
- Chandrasekharaiah, D.S. and Srinath, K.(1998). "Thermoelastic interactions without energy dissipation due to a point heat source", *J. Elast.* **50(2)**, 97-108.
- Chapman, C.(2004). *Fundamentals of Seismic Wave Propagation*, Cambridge University Press, New York.
- Chattopadhyay, A. and Choudhury, S.(1990). "Propagation, reflection and transmission of magnetoelastic shear waves in a self-reinforced medium", *Int. J. Eng. Sci.* **28**, 485–495.
- Chattopadhyay, A., Venkateswarlu, R.L.K. and Saha, S.(2002). "Reflection of quasi-P and quasi-SV waves at the free and rigid boundaries of a fibre-reinforced medium", *Sadhana.* **27**, 613–630.
- Chaudhary, S., Kaushik, V.P. and Tomar, S.K.(2004). "Reflection/transmission of plane wave through a self-reinforced elastic layer between two half-spaces", *Acta Geophys. Pol.* **52**, 219–235.
- Chaudhuri, S.R. and Debnath, L.(1983). "Magneto-thermo-elastic plane waves in rotating media", *Int. J. Eng. Sci.* **21(2)**, 155-163.
- Chirita, S.(1979). "Existence and Uniqueness Theorems for Linear Coupled Thermoelasticity with Microstructure", *J. Therm. Stresses.* **2**, 157–169.
- Ciarletta, M.(1999). "On the bending of microstretch elastic plates". *Int. J. Eng. Sci.* **37(10)**, 1309-1318.
- Ciarletta, M., Svanadze, M. and Buonanno, L.(2009). "Plane waves and vibrations in the theory of micropolar thermoelasticity for materials with voids", *Eur. J. Mech. A-Solid* **28(4)**, 897-903.

- Cosserat, E.M.P. and Cosserat, F.(1909) *Theory of deformable bodies*, Paris, A. Hermann and sons.
- Dai, H.L. and Wang X.(2006). "Magnetothermostress wave propagation and perturbation of magnetic field vector in an orthotropic laminated hollow cylinder", *J. Therm. Stresses*. **29(8)**, 791-806.
- Day, W.A(1985). *Heat Conduction Within Linear Thermoelasticity*, Springer, New Work.
- Debnath, S. and Singh, S.S.(2025). "Effect of thermal diffusivity on Rayleigh waves in human tissue", *J. Vib. Control*. **31(5-6)**, 1022-1033.
- Deswal, S., Sheokand S.K. and Kalkal K.K.(2019). "Reflection at the free surface of fiber-reinforced thermoelastic rotating medium with two-temperature and phase-lag", *Appl. Math. Model*. **65**, 106–119.
- Dhabu, A.C. and Raghukanth S.T.G.(2019). "Fundamental solutions to static and dynamic loads for homogeneous reduced micropolar half-space", *Pure Appl. Geophys*. **176**, 4881–4905.
- Dhaliwal, R.S. and Sherief, H.H.(1980). "Generalized thermoelasticity for anisotropic media", *Q. Appl. Math*. **38(1)**, 1-8.
- Dost, S. and Tabarrok, B.(1978). "Generalized micropolar thermoelasticity", *Int. J. Eng. Sci*. **16(3)**, 173-183.
- El-Karamany, A.S. and Ezzat, M.A.(2004). "Analytical aspects in boundary integral equation formulation for the generalized linear micropolar thermoelasticity", *Int. J. Mech. Sci*. **46(3)**, 389-409.
- Eringen, A.C.(1966). "Linear Theory of Micropolar Elasticity", *J.Math. Mech*. **15(6)**, 909-923.

- Eringen, A.C.(1990). "Theory of thermo-microstretch elastic solids", *Int. J. Eng. Sci.* **28(12)**, 1291-1301.
- Eringen, A.C. and Suhubi, E.(1964). "Nonlinear theory of simple micro-elastic solids—I", *Int. J. Eng. Sci.* **2(2)**, 189-203.
- Ewing, W.M., Jardetsky, W.S. and Press, F.(1957). *Elastic Waves in Layered Media*, McGraw-Hill, New York.
- Ezzat M.A.(2004). "Fundamental solution in generalized magneto-thermoelasticity with two relaxation times for perfect conductor cylindrical region", *Int. J. Eng. Sci.* **42(13-14)**, 1503-1519.
- Ezzat, M.A., Abbas, I.A., El-Bary, A.A. and Ezzat, S.M.(2014). "Numerical study of the Stokes' first problem for thermoelectric micropolar fluid with fractional derivative heat transfer", *Magnetohydrodynamics.* **50(3)**, 263-277.
- Ezzat, M.A. and Awad, E.S.(2009). "Micropolar generalized magneto-thermoelasticity with modified Ohm's and Fourier's laws", *J. Math. Anal. Appl.* **353(1)**: 99-113.
- Ezzat, M.A. and Elall, M.Z.(2009). "Generalized magneto-thermoelasticity with modified Ohm's law", *Mech. Adv. Mater. Struct.* **17(1)**, 74-84.
- Ezzat, M.A., Othman M.I. and El-Karamany A.S.(2001). "Electromagneto-thermoelastic plane waves with thermal relaxation in a medium of perfect conductivity", *J. Therm. Stresses.* **24(5)**, 411-432.
- Ezzat, M.A. and Youssef, H.M.(2005). "Generalized magneto-thermoelasticity in a perfectly conducting medium", *Int. J. Solids Struct.* **24-25**, 6319-6334.
- Farhan, A.M. and Abd-Alla, A.M.(2018). "Effect of rotation on the surface wave propagation in magneto-thermoelastic materials with voids", *J. Ocean Eng. Sci.* **3(4)**, 334-342.

- Goyal, S., Singh, D. and Tomar, S.K.(2016). "Rayleigh-type surface waves in a swelling porous half-space", *Transp. Porous Media.* **113**, 91-109.
- Goyal, S. and Tomar, S.K.(2015). "Reflection/refraction of a dilatational wave at a plane interface between uniform elastic and swelling porous half-spaces", *Transp. Porous Media.* **109(3)**, 609-632.
- Graff, K.F.(1991). *Wave Motion in Elastic Solids*, Dover Publications, United States.
- Green, A.E. and Adkins J. E.(1960). *Large Elastic Deformations and Non-linear Continuum Mechanics*, Clarendon Press, United Kingdom.
- Green A.E. and Laws N.(1972). "On a global entropy production inequality", *Q. J. Mech. Appl. Math.* **25(1)**, 1-11.
- Green, A.E. and Lindsay, K.(1972). "Thermoelasticity", *J. Elast.* **2(1)**, 1-7.
- Green, A.E. and Naghdi, P.M.(1991). "A re-examination of the basic postulates of thermomechanics", *Proc. R. Soc. Lond.:A* **432**, 171-194.
- Green, A.E. and Naghdi, P.(1992). "On undamped heat waves in an elastic solid", *J. Therm. Stress.* **15(2)**, 253-264.
- Green, A.E. and Naghdi, P.(1993). "Thermoelasticity without energy dissipation", *J. Elast.* **31(3)**, 189-208.
- Gurijala, R., Ala, S. and Perati, M.R.(2023). "Analysis of reflection and transmission of axially-symmetric waves for self-reinforced half space over a poroelastic half space", *J. Theor. Appl. Mech.* **53**, 3-18.
- Hanyga, A.(1985). *Seismic Wave Propagation in the Earth*, Elsevier Science Ltd, Warszawa.
- Hetnarski, R.B.(2014). *Encyclopedia of thermal stresses*, Springer, Netherlands.

- Hetnarski, R.B. and Ignaczak, J.(1999). "Generalized thermoelasticity", *J. Therm. Stresses*. **4**, 451-476.
- Hilal, M.I.M., Elaziz, E.M.A. and Hanoura, S.A.(2021) "Reflection of plane waves in magneto-micropolar thermoelastic medium with voids and one relaxation time due to gravity and two-temperature theory", *Indian J. Phys.* **95**, 915–924.
- Iesan, D. and Pompei, A.(1995). "On the equilibrium theory of microstretch elastic solids", *Int. J. Eng. Sci.* **33(3)**, 399-410.
- Iesan, D. and Scalia, A.(1996). *Thermoelastic Deformations*, Springer, New Work.
- Ignaczak, J.(1979). "Uniqueness in generalized thermoelasticity", *J. Therm. Stresses*. **2(2)**, 171-175.
- Ismail, G.M., Gepreel, K.A., Lotfy, K., Mahdy, A.M.S., El-Bary, A. and Saeed A.M.(2022). "Influence of variable thermal conductivity on thermal-plasma-elastic waves of excited microelongated semiconductor", *Alex. Eng. J.* **61(12)**, 12271-12282.
- Jatain, S., Deswal, S. and Kalkal, K.K.(2023). "Thermodynamical interactions in a micropolar magneto-thermoelastic medium with photothermal effect", *Int. J. Numer. Methods Fluids*. **33(11)**, 3642-3668.
- Kalkal, K.K., Sheoran, D. and Deswal, S.(2020). "Reflection of plane waves in a nonlocal micropolar thermoelastic medium under the effect of rotation", *Acta Mech.* **231**, 2849–2866.
- Kaur, J. and Tomar, S.K.(2004). "Reflection and refraction of SH-waves at a corrugated interface between two monoclinic elastic half-spaces", *Int. J. Numer. Anal. Methods Geomech.* **28(15)**, 1543-1575.
- Khan A.A., Sait S.M. and Ellahi R.(2025). "Reflection of Harmonic Waves in a Nonlocal Rotating Micropolar Medium with Constant Magnetic Field un-

- der Three-phase-lag Theory with Temperature Dependent Elastic Model", *J. Comput. Appl. Mech.* **56(2)**, 331-344.
- Knopoff, L.(1955). "The interaction between elastic wave motions and a magnetic field in electrical conductors", *J. Geophys. Res.* **60(4)**, 441-456.
- Kolsky, H.(1963). "Stress waves in anelastic solids", *J. Geophys. Res.* **68(4)**, 1193-1194.
- Kumar, A. and Tomar S.K.(2022). "Coupled dilatational waves at a plane interface between two dissimilar magneto-elastic half-spaces containing voids", *Acta Mech.* **233**, 5061-5087.
- Kumar, M., Barak, M.S. and Kumari, M.(2019). "Reflection and refraction of plane waves at the boundary of an elastic solid and double-porosity dual-permeability materials", *Pet. Sci.* **16**, 298–317.
- Kumar, M. and Saini, R.(2012). "Reflection and refraction of attenuated waves at the boundary of elastic solid and porous solid saturated with two immiscible viscous fluids", *Appl. Math. Mech.(Engl. Ed.)*. **33(6)**, 797–816.
- Kumar, M. and Saini, R.(2016). "Reflection and refraction of waves at the boundary of non viscous porous solid saturated with single fluid and a porous solid saturated with immiscible fluids", *Lat. Am. J. Solids Struct.* **13**, 1299–1324.
- Kumar, R.(2000). "Wave propagation in a micropolar viscoelastic generalized thermo-elastic solid", *Int. J. Eng. Sci.* **38**, 1377–1395.
- Kumar, R. and Abbas, I.A.(2013). "Deformation due to thermal source in micropolar thermoelastic media with thermal and conductive temperatures", *J. Comput. Theor. Nanosci.* **10(9)**, 2241–2247.

- Kumar, R. and Ailawalia, P.(2007). "Inclined load in micropolar thermoelastic medium possessing cubic symmetry with one relaxation time", *Mech. Mech. Eng.* **11**, 129–157.
- Kumar, R. and Deswal, S.(2001). "Mechanical and Thermal Source in a Micropolar Generalized Thermoelastic Medium", *J. Sound Vib.* **239**, 467–488.
- Kumar, R. and Deswal S.(2002). "Surface wave propagation in a micropolar thermo-elastic medium without energy dissipation", *J. Sound Vib.* **256**, 173–178.
- Kumar, R., Devi, S. and Abo-Dahab, S.M.(2018). "Stoneley waves at the boundary surface of modified couple stress generalized thermoelastic with mass diffusion", *J. Appl. Sci. Eng.* **21(1)**, 1-8.
- Kumar, R. and Gupta, R.R.(2010). "Propagation of waves in transversely isotropic micropolar generalized thermoelastic half space", *Int. Commun. Heat Mass Trans.* **37**, 1452-1458.
- Kumar, R., Kaushal, S. and Marin, M.(2018). "Propagation of waves in micropolar thermodiffusion elastic half-space", *Afr. Mat.* **29(3–4)**, 451–462.
- Kumar R., Vohra R. and Gorla M.G. (2016). "Some considerations of fundamental solution in micropolar thermoelastic materials with double porosity", *Arch. Mech.* **68(4)**, 263–284
- Kumar, S., Kadian, A. and Kalkal, K.K.(2022). "Dual-phase-lag model for a non-local micropolar thermoelastic half-space subjected to gravitational field and inclined load", *Int. J. Numer. Methods Heat Fluid Flow.* **32(6)**, 1999-2026.
- Kumar, S. and Partap, G.(2023). "Analysis of waves in micropolar generalized thermoelastic plate with memory dependent derivatives", *ZAMM* **103(2)**.

- Kundu, S., Kalkal, K.K., Sangwan, M. and Sheora, D.(2023). "Two-dimensional deformations in an initially stressed nonlocal micropolar thermoelastic porous medium subjected to a moving thermal load", *Int. J. Numer. Methods Heat Fluid Flow.* **33(3)**, 1116-1143.
- Kumar, V. and Nazir, R.(2023). "A study of thermo-mechanical interactions in the rotating micropolar elastic solid with two temperatures using memory-dependent derivative", *Mech. Solids.* **58(1)**, 325-337.
- Lata, P.(2018). "Reflection and refraction of plane waves in a layered nonlocal elastic and anisotropic thermoelastic medium", *Struct. Eng. Mech.* **66(1)**, 113-124.
- Lianngenga, R., Singh S.S.(2020). "Reflection of coupled dilatational and shear waves in the generalized micropolar thermoelastic materials", *J. Vib. Control.* **26(21-22)**, 1948-1955.
- Lindsay, R.B.(1960). *Mechanical radiation*, New York, McGraw-Hill Book Company.
- Lord, H.W. and Shulman Y.(1967). "A generalized dynamical theory of thermoelasticity", *J. Mech. Phys. Solids.* **15(5)**, 299-309.
- Lotfy, K., Abo-Dahab, S.M. and Hobiny, A.D.(2019). "Magneto-rotation-fibre-reinforced thermoelastic with gravity and energy dissipation", *Int. J. Comput. Methods Eng. Sci. Mech.* **20(1)**, 14-28.
- Lotfy, K. and El-Bary, A.A.(2022). "Magneto-photo-thermo-microstretch semiconductor elastic medium due to photothermal transport process", *Silicon.* **14(9)**, 4809-4821.
- Lotfy, K., El-Bary A.A. and El-Sharif A.H.(2020). "Ramp-type heating microtemperature for a rotator semiconducting material during photo-excited processes with magnetic field", *Results Phys.* **19**, 103338.

- Lotfy, K., El-Bary, A.A., Hassan, W. and Ahmed, M.H.(2020). "Hall current influence of microtemperature magneto-elastic semiconductor material", *Superlattices Microstruct.* **139**, 106428.
- Lotfy, K., Elidy, E.S. and Tantawi, R.S.(2021). "Photothermal excitation process during hyperbolic two-temperature theory for magneto-thermo-elastic semiconducting medium", *Silicon.* **13(7)**, 2275-2288.
- Love, A.E.H.(1911). *Some Problems of Geodynamics*, Cambridge University Press, London.
- Love, A.E.H.(1944). *A Treatise on the Mathematical Theory of Elasticity*, Dover Publications, New York.
- Mahdy, A.M.S., Lotfy, K., Ismail, E.A., El-Bary, A., Ahmed, M. and El-Dahdouh, A.A.(2020) "Analytical solutions of time-fractional heat order for a magneto-photothermal semiconductor medium with Thomson effects and initial stress", *Results Phys.* **18**, 103174.
- Markov, M.(2009). "Reflection of elastic waves at an interface between two porous half-spaces filled with different fluids", *Izv. Phys. Solid Earth.* **45(9)**, 769–776.
- Markov, M., Markova, I. and Pervago, E.(2019). "Reflection and transmission of elastic waves by a fluid-filled crack located in a poroelastic formation", *J. Appl. Geophys.* **167**, 63–72.
- Maxwell, J.C.(1861). "On physical lines of force", *Philosophical Magazine.* **90**, 11–23.
- Maxwell, J.C.(1862) "III. On physical lines of force", *The Philosophical Magazine: A Journal of Theoretical Experimental and Applied Physics* **23**, 12-24.
- McCarthy, M.F.(1972). "Wave propagation in generalized thermoelasticity", *Int. J. Eng. Sci.* **10(7)**, 593-602.

- Miklowitz, J.(1966). *Elastic wave propagation*, Spaniel Publications, London. Applied Mechanics Surveys, Spartan Books.
- Mindlin, R.D.(1964). "Microstructure in Linear Elasticity", *Arch. Rat. Mech. Anal.* **16**, 51-78.
- Mindlin, R.D. and Tiersten, H.(1962). "Effects of couple-stresses in linear elasticity", *Arch. Ration. Mech. Anal.* **11(1)**, 415-448.
- Moncef, A.(2009). "Theory of generalized micropolar thermoe-lastic diffusion under Lord-Shulman model", *J. Therm. Stresses.* **32(9)**, 923–942.
- Morse, P.M. and Ingard, K.U.(1968) *Theoretical acoustics*, New York, McGraw-Hill Book Company.
- Nath, S., Sengupta, P.R. and Debnath, L.(1998). "Magneto-thermoelastic surface waves in micropolar elastic media", *Comput. Math. Appl.* **35(3)**, 47-55.
- Nayfeh, A.H.(1995). *Wave propagation in layered anisotropic media with applications to composites*, Elsevier, Netherlands.
- Nayfeh, A. and Nemat-Nasser, S.(1971). "Thermoelastic waves in solids with thermal relaxation", *Acta Mech.* **1**, 53-69.
- Neff, P., Jeong, J., Münch, I. and Ramézani, H.(2009). "Mean field modeling of isotropic random Cauchy elasticity versus microstretch elasticity", *Z. Angew. Math. Phys.* **60**, 479-497.
- Nesarhosseini, S., Ansari R., Faraji Oskouie, M. and Rouhi, H.(2023) "Thermally induced vibration analysis of Timoshenko beams based on the micropolar thermoelasticity", *Acta Mech.* **234**, 1957–1971.
- Nowacki, W.(1970). "The Plane Problem of Micropolar Thermoelasticity", *Arch. Mech.* **22**, 3–26.

- Nowacki, W.(1971). "Two-dimensional problem magnetoelasticity", *Bull. Acad. Polon. Sci. Tech*, **19(4)**, 307-311.
- Nowacki, W.(1975). *Dynamic problems of thermoelasticity*, Noordhoff International Publishing, Netherlands.
- Ohm, G.S.(1827). *Die Galvanische Kette*, Bie T.H. Riemann, Berlin.
- Othman, M.I.A.(2004). "Effect of rotation on plane waves in generalized thermoelasticity with two relaxation times", *Int. J. Solids Struct.* **41(11-12)**, 2939-2956.
- Othman, M.I.A. and Abbas, I.A.(2012). "Generalized thermoelasticity of thermal-shock problem in a non-homogeneous isotropic hollow cylinder with energy dissipation", *Int. J. Thermophys.* **33(5)**, 913-923.
- Othman, M.I.A. and Atwa, S.Y.(2012). "Response of micropolar thermoelastic solid with voids due to various sources under Green Naghdi theory", *Acta Mech. Solida Sin.* **25(2)**, 197-209.
- Othman, M.I.A., Khan, A., Jahangir, R. and Jahangir, A.(2019). "Analysis on plane waves through magneto-thermoelastic microstretch rotating medium with temperature dependent elastic properties", *Appl. Math. Model.* **65**, 535-548.
- Othman, M.I.A. and Kumar, R.(2009). "Reflection of magneto-thermoelasticity waves with temperature dependent properties in generalized thermoelasticity", *Int. Commun. Heat Mass Transf.* **36(5)**, 513-20.
- Othman, M. and Lotfy, K.(2009). "Transient disturbance in a half-space under generalized magneto-thermoelasticity with internal heat source", *Acta Phys. Pol. A.* **116(2)**, 185-192.
- Othman, M.I.A. and Mondal, S.(2020). "Memory-dependent derivative effect on wave propagation of micropolar thermoelastic medium under pulsed laser heat-

- ing with three theories", *Int. J. Numer. Methods Heat Fluid Flow.* **30(3)**, 1025–1046.
- Othman, M.I.A. and Singh, B.(2007). "The effect of rotation on generalized micropolar thermoelasticity for a half-space under five theories", *Int. J. Solids Struct.* **44(9)**, 2748–2762.
- Othman, M.I.A. and Song, Y.G.(2006). "The effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation", *Acta Mech.* **184(1)**, 189-204.
- Othman, M.I.A. and Song, Y.(2008). "Reflection of magneto-thermo-elastic waves from a rotating elastic half-space", *Int. J. Eng. Sci.* **46(5)**, 459-474.
- Parfitt, V.C. and Eringen, A.C.(1969). "Reflection of plane waves from the flat boundary of a micropolar elastic half-space", *J. Acoust. Soc. Am.* **45**, 1258–1272.
- Paria, G.(1966). "Magneto-elasticity and magneto-thermo-elasticity", *Adv. Appl. Mech.* **10**, 73-112.
- Passarella, F.(1996). "Some Results in Micropolar Thermoelasticity", *Mech. Res. Commun.* **23**, 349–357.
- Perati, M.R., Ala, S. and Gurijala, R.(2019). "Study of reflection and transmission of axially symmetric body waves incident on a base of semi-infinite poroelastic solid cylinder", *Arch. Appl. Mech.* **89(12)**, 2507-2517.
- Pujol, J.(2003). *Elastic wave propagation and generation in seismology*, Cambridge University Press, New York.
- Puri, P.(1973). "Plane waves in generalized thermoelasticity", *Int. J. Eng. Sci.* **7**, 735-744.

- Payton, R.G.(1983). *Elastic wave propagation in transversely isotropic media*, Springer, Netherlands.
- Rayleigh, L.(1885). "On waves propagated along the plane surface of an elastic solid", *Proc. London Math. Soc.* **17**, 4-11.
- Rayleigh, L.(1945). *The theory of sound, Vols. I and II*, Dover Publications, USA.
- Roy, I., Acharya, D.P. and Acharya, S.(2017). "Propagation and reflection of plane waves in a rotating magneto-elastic fibre-reinforced semi space with surface stress", *Mech. Mech. Eng.* **21(4)**, 1043–1061.
- Sahrawat, R.K.(2021). "Reflection-refraction coefficients and energy ratios in couple stress micropolar thermoviscous elastic solid", *Int. J. Appl. Mech. Eng.* **26(2)**, 47–65.
- Said, S.M., Elmaklizi, Y.D. and Othman M.I.A.(2017). "A two-temperature rotating-micropolar thermoelastic medium under influence of magnetic field", *Chaos Solit. Fractals.* **97**, 75–83.
- Sargsyan, S.H.(2012). "Mathematical Model of Micropolar Elastic Thin Plates and Their Strength and Stiffness Characteristics", *J. Appl. Mech. Tech. Phys.* **53(2)**, 275–282.
- Sarkar, N.(2014). "Generalized magneto-thermoelasticity with modified Ohm's law under three theories", *Comput. Math. Model.* **25(4)**, 544-564.
- Sarkar, N. and De, S.(2020). "Waves in magneto-thermoelastic solids under modified Green–Lindsay model", *J. Therm. Stress.* **43(5)**, 594-611.
- Sarkar, N. and Lahiri, A.(2012). "A three-dimensional thermoelastic problem for a half-space without energy dissipation", *Int. J. Eng. Sci.* **51**, 310-325.
- Sato, H. and Fehler, M.C.(2009). *Seismic Wave Propagation and Scattering in the Heterogeneous Earth*, Springer, New Work.

- Sharma, J.N. and Chand, D.(1988). "Transient generalised magnetothermoelastic waves in a half-space", *Int. J. Eng. Sci.* **9**, 951-958.
- Sharma, J.N., Kumar V., and Chand D.(2003). "Reflection of generalized thermoelastic waves from the boundary of a half-space", *J. Therm. Stress.* **26**, 925-942.
- Shanker, M. and Dhaliwal, R.(1975). "Dynamic Coupled Thermoelastic Problems in Micropolar Theory", *Int. J. Eng. Sci.* **13**, 121-148.
- Shekhar, S. and Parvez, I.A.(2016). "Reflection and refraction of attenuated waves at the interface between cracked poroelastic medium and porous solid saturated with two immiscible fluids", *Transp. Porous Media.* **113(2)**, 405-430.
- Sherief, H.H.(1993). "State space formulation for generalized thermoelasticity with one relaxation time including heat sources", *J. Therm. Stress.* **16(2)**, 163-180.
- Sherief, H.H., Hamza F.A. and El-Sayed, A.M.(2005). "Theory of generalized micropolar thermoelasticity and an axi-symmetric half-space problem", *J. Therm. Stress.* **28(4)**, 409-437.
- Sherief, H.H. and Helmy, K.A.(2002). "A two-dimensional problem for a half-space in magneto thermoelasticity with thermal relaxation", *Int. J. Eng. Sci.* **5**, 587-604.
- Singh, B.(2000). "Reflection of plane sound wave from a micropolar generalized solid half-space", *J. Sound Vib.* **235**, 685-696.
- Singh, B.(2009). "Wave propagation in a generalized thermoelastic material with voids", *Appl. Math. Comput.* **189(1)**, 698-709.
- Singh, B.(2017). "Reflection of elastic waves from plane surface of a half-space with impedance boundary conditions", *Geosci. Res.* **2(4)**, 242-253.

- Singh, S.S., Debnath S. and Othman M.I.(2022). "Thermoelastic Theories on the Refracted Waves in Microstretch Thermoelastic Diffusion Media", *Int. J. Appl. Mech.* **14(2)**: 2250008.
- Singh, B. and Kumar, R.(1998a). "Reflection of plane wave from a flat boundary of micropolar generalized thermoelastic half-space", *Int. J. Eng. Sci.* **36**, 865–890.
- Singh, B. and Kumar, R.(1998b). "Reflection and refraction of micropolar elastic waves at a loosely bonded interface between viscoelastic solid and micropolar elastic solid", *Int. J. Eng. Sci.* **36**, 101–117.
- Singh, B. and Pal, R.(2011). "Surface wave propagation in a generalized thermoe-
lastic material with voids", *Appl. Math.* **2(5)**, 521.
- Singh, S.S.(2013). "Transverse wave at a plane interface in thermo-elastic materials with voids", *Meccanica.* **48**, 617-630.
- Singh, B. and Singh, S.J.(2004). "Reflection of plane waves at the free surface of a fibre-reinforced elastic half-space", *Sadhana.* **29(3)**, 249–257.
- Singh, S.S. and Tothhawng, L.(2019). "Stoneley and Rayleigh waves in thermoe-
lastic materials with voids", *J. Vib. Control.* **25(14)**, 2053-2062.
- Singh, S.S. and Tomar, S.K.(2007). "Shear waves at a corrugated interface between two dissimilar fiber-reinforced elastic half-spaces", *J. Mech. Mater. Struct.* **2(1)**, 167-188.
- Singh, S.S., Zorammuana, C. and Singh, B.(2014). "Elastic waves at a plane interface between two dissimilar incompressible transversely isotropic fibre-reinforced elastic half-spaces", *Int. J. Appl. Math. Sci.* **7(2)**, 131-146.
- Sinha, A.N. and Sinha, S.B.(1974). "Reflection of thermoelastic waves at a solid half-space with thermal relaxation", *J. Phys. Earth.* **22**, 237-244.

- Sinha, S.B. and Elsibai, K.A.(1996). "Reflection of thermoelastic waves at a solid half-space with two thermal relaxation times", *J. Therm. Stress.* **19(8)**, 763–777.
- Sinha, S.B. and Elsibai, K.A.(1997). "Reflection and refraction of thermoelastic waves at an interface of two semi-infinite media with two thermal relaxation times", *J. Therm. Stress.* **20**, 129–146.
- Sokolnikof, I.S.(1946). *Mathematical theory of elasticity*, McGraw-Hill Book Company, New York.
- Song, Y.G., Zhang, Y.C., Xu, H.Y. and Lu, B.H(2004). "Reflection of magneto-thermo-viscoelastic wave under generalized thermoviscoelasticity", *Int. J. Thermophys.* **25**, 909–929.
- Tauchert, T.R., Claus Jr, W.D. and Ariman, T.(1968). "The linear theory of micropolar thermoelasticity", *Int. J. Eng. Sci.* **6(1)**, 37-47.
- Tolstoy, I. and Clay, C.S.(1966). *Ocean acoustics*, New York, McGraw-Hill Book Company.
- Tomar, S.K. and Arora, A.(2006). "Reflection and transmission of elastic waves at an elastic/porous solid saturated by two immiscible fluids", *Int. J. Solids Struct.* **43**, 1991–2013.
- Tomar, S.K. and Gogna, M.L.(1992). "Reflection and refraction of a longitudinal microrotational wave at an interface between two micropolar elastic solids in welded contact", *Int. J. Eng. Sci.* **30(11)**, 1637-1646.
- Tomar, S.K. and Gogna, M.L.(1995). "Reflection and refraction of longitudinal wave at an interface between two micropolar elastic solids in welded contact", *J. Acoust. Soc. Am.* **97(2)**, 822-830.

- Tomar, S.K. and Gogna, M.L.(1995). "Reflection and refraction of longitudinal wave at an interface between two micropolar elastic solids in welded contact", *J. Acoust. Soc. Am.* **97(2)**, 822-830.
- Tomar, S.K. and Khurana, A.(2008). "Transmission of longitudinal wave at plane interface between micropolar elastic and chiral solid half-spaces incident from micropolar half-spaces", *J. Sound Vib.* **311**, 973-990.
- Tomar, S.K. and Singh, D.(2006a). "Propagation of Stoneley waves at an interface between two microstretch elastic half-spaces", *J. Vib. Control.* **12(9)**, 995-1009.
- Tomar, S.K. and Singh, S.S.(2006b). "Plane SH-waves at a corrugated interface between two dissimilar perfectly conducting self-reinforced elastic half-spaces", *Int. J. Numer. Anal. Methods Geomech.* **30(6)**, 455-487.
- Tung, D.X.(2021). "The reflection and transmission of a quasi-longitudinal displacement wave at an imperfect interface between two nonlocal orthotropic micropolar half-spaces", *Arch. Appl. Mech.* **91(10)**, 4313-4328.
- Tzou, D.Y.(1995). "A unified field approach for heat conduction from macro-to micro-scales", *J. Heat Transfer.* **117(1)**, 8-16.
- Udias, A.(1999). *Principles of Seismology*, Cambridge University Press, United Kingdom. **27(3)**, 240-53.
- Wang, E., Carcione, J.M., Ba, J. and Liu, Y.(2020). "Reflection and transmission of plane elastic waves at an interface between two double-porosity media: effect of local fluid flow", *Surv. Geophys.* **41(2)**, 283-322.
- Wang, X. and Dai, H.L.(2004). "Magnetothermodynamic stress and perturbation of magnetic field vector in an orthotropic thermoelastic cylinder", *Int. J. Eng. Sci.* **5**, 539-556.

- Wei, W., Zheng, R., Liu, G. and Tao, H.(2016) "Reflection and refraction of P wave at the interface between thermoelastic and porous thermoelastic medium", *Transp. Porous Media.* **113**(1), 1–27.
- Yadav, A.K.(2021). "Thermoelastic waves in a fractional-order initially stressed micropolar diffusive porous medium", *J. Ocean Eng. Sci.* **6**(4), 376–388.
- Youssef, H.M.(2005). "Generalized thermoelasticity of an infinite body with a cylindrical cavity and variable material properties", *J. Therm. Stresses.* **5**, 521-532.
- Youssef, H.M.(2006). "Theory of two-temperature-generalized thermoelasticity", *IMA J. Appl. Math.* **71**(3), 383-390.
- Youssef, H.M.(2010). "Theory of fractional order generalized thermoelasticity", *ASME J. Heat Mass Transf.* **132**(6), 061301.
- Youssef, H.M.(2011). "Theory of two-temperature thermoelasticity without energy dissipation", *J. Therm. Stresses.* **34**(2), 138-146.
- Yu, Y.J., Tian, X.G. and Lu, T.J.(2013). "Fractional order generalized electro-magneto-thermo-elasticity", *Eur. J. Mech. A/Solids.* **42**, 188-202.
- Zakaria, M.(2012). "Effects of Hall Current and Rotation on Magneto-Micropolar Generalized Thermoelasticity Due to Ramp-Type Heating", *Int. J. Appl. Electromagn. Mech.* **2**(3), 24–32.
- Zhou, D., Yin, X. and Zong, Z.(2019). "The characteristics of reflection and transmission coefficients of porous medium saturated with an ideal fluid", *Ann. Geophys.* **62**(5), 1–9 .
- Zorammuana, C. and Singh, S.S.(2016). "Elastic waves in thermoelastic saturated porous medium", *Meccanica.* **51**(3), 593-609.
- Zorammuana, C., and Singh, S.S.(2017). "Effect of porosity on wave propagation in thermoelastic saturated porous half-spaces", *Asian J. Math..* **16**(2), 96-112.

Zorammuana, C. and Singh, S.S.(2022). "Magnetic and rotational effects on the reflection of inhomogeneous waves in a fiber-reinforced thermoelastic material", *Mech. Adv. Mater. Struct.* **29(28)**, 7690-7702.

Zorammuana, C. and Singh, S.S.(2024). "Effect of magnetic field on the reflection/transmission of inhomogeneous waves in fiber-reinforced thermoelastic solids", *Waves Random Complex Media.* **34(4)**, 2919-2944.

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ABSTRACT

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**DEPARTMENT OF MATHEMATICS AND
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ABSTRACT

**A STUDY OF ELASTIC WAVES IN
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BY

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Submitted

**In partial fulfillment of the requirement of the Degree of Doctor of
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ABSTRACT

The subject of wave and vibrations in different elastic continuum has been a matter of concern for many researchers since long. It is used in various field like in the exploration of mines and petroleum, earthquake engineering, Seismology, Geophysics, etc. More specific, wave phenomena in thermoelasticity materials has wide applications in engineering. The following objectives are taken up in the thesis:

1. Reflection of elastic waves from a half-space of electro magneto thermoelastic solid.
2. 3D Analysis of Wave Propagation in Generalized Magneto Thermoelastic Medium.
3. Reflection of elastic waves from a half-space of micropolar generalized magneto-thermoelastic materials.
4. Refraction of P and SV -waves from a plane interface in micropolar generalized magneto-thermoelastic half-spaces.

The first chapter is the general introduction of the thesis which includes the fundamental concepts of base vectors, continuum concept, elastic waves, reflection and refraction at plane interface, thermoelasticity, Maxwell's Equations, Modified Ohm's law, constitutive relation and review of relevant literature.

Chapter 2, investigates the problem of reflection of homogeneous plane waves from the free surface of a generalized magneto thermoelastic half-space. The phase speeds of corresponding to longitudinal, transverse and thermal waves are obtained and are independent of angle of propagation. The amplitude and energy ratios corresponding to the reflected waves are obtained theoretically and numerically with the help of boundary conditions. The effects of magnetic and thermal parameters on the reflected waves are examined. The results hold the conservation law of energy. Various special cases are derived for this problem. We have observed that the present study confirms the significant effects of the magnetic parameter and thermal parameter on the wave propagation. The following points are the concluding remarks:

- I. The amplitude and energy ratios are functions of angle of incidence, magnetic, thermal and elastic parameters.

- II. The phase velocity of longitudinal wave increases with the increase of H_0 and C_e .
- III. The phase velocity corresponding to thermal wave increases with the increase of ω .
- IV. The variations of Z_1 and Z_2 are similar in nature.
- V. The effect of H_0 on Z_i is very small.
- VI. The effects of H_0 and C_e on E_i are minimum near the grazing angle of incidence.
- VII. The sum of energy ratios of reflected waves are close to one.

Third chapter studied the three dimensional problem of wave propagation in generalized magneto thermoelastic medium. The eigenvalue approach is adopted to solve the vector matrix differential equation which represents the equation of motion for the generalized thermoelastic materials. Components of displacements and temperature field are analyzed theoretically and computed numerically with the help of appropriate boundary conditions. The effect of magnetic and relaxation time on the propagation of waves are critically examined. The effects of H_0 and τ_0 on displacement, stress tensors and temperature field are depicted graphically. We can conclude the following points:

- I. The effect τ_0 on the displacement and stress is very small near origin. The effect of H_0 on the components of displacement and temperature field is also very small.
- II. The values of σ_{xz} and σ_{yz} increase with the increase of z .
- III. The values of σ_{xz} and σ_{yz} increase with the increase of H_0 and τ_0 .
- IV. The values of u, v, w and σ_{zz} decrease with the increase of H_0 and τ_0 .
- V. Similar natures are observed for u, w and σ_{zz} .
- VI. Similar natures are observed for v and T .

Chapter 4, discusses the problem of reflection of homogeneous plane waves from the free surface of generalized magneto micropolar thermoelastic half-space using

modified Ohm's law and generalized Fourier law. There are five waves propagated in the medium and these waves are longitudinal, transverse, micropolar, thermal and another wave due to magnetic field. The phase velocities of these waves are obtained analytically and numerically. Using appropriate boundary conditions, the amplitude and energy ratios corresponding to the reflected waves are obtained analytically and numerically. Magnetic and thermal effects on the reflected waves are investigated and found the conservation law of energy in all cases. The following points are concluded from the analysis:

- I. The values of phase velocity corresponding to V_i , ($i = 1, 3, 4, 5$) decrease with the increase of C_e .
- II. The values of phase velocity, V_2 increase with the increase of C_e .
- III. The values of amplitude ratios, Z_1 , Z_2 and Z_5 decrease with the increase of H_0 and C_e .
- IV. The values of amplitude ratios, Z_3 and Z_4 increase with the increase of H_0 and C_e . The behaviour of energy ratios, E_i of the refracted waves are similar with Z_i .
- V. The sum of energy ratios of the reflected waves are close to unity for different values of H_0 and C_e .

Chapter 5 investigate a model of reflection/refraction of homogeneous plane waves at a plane interface of generalized magneto micropolar thermoelastic half-spaces using modified Ohm's law and generalized Fourier law. The phase velocities of these waves are obtained analytically and numerically. The amplitude and energy ratios corresponding to the reflected and refracted waves are obtained analytically and numerically with the use of suitable boundary conditions. The effects of magnetic parameters on the amplitude and energy ratios are examined. The conservation of energy through the reflected and refracted waves verify the problem. The following points are concluded from the analysis:

- I. The values of amplitude ratios for reflected waves, Z_1 , Z_2 , Z_3 and Z_4 increase with the increase of H_0 and Z_5 decreases with the increase of H_0 .
- II. The values of amplitude ratios for refracted waves, Z_i ($i = 6, 7, 8, 9, 10$) decrease with the increase of H_0 .

III. The effects of H_0 on E_i are minimum near the grazing angle of incidence.

The behaviour of energy ratios, E_i of the reflected/refracted waves are similar with Z_i .

IV. The sum of the all energy ratios are close to unity for different values of H_0 .

Chapter 6 gives the summary and conclusion of the thesis.

Finally, the list of references is given at the end.